

STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2011-12 & thereafter)

SUBJECT CODE : 11MT/MC/AG24

B. Sc. DEGREE EXAMINATION, APRIL 2015
BRANCH I – MATHEMATICS
SECOND SEMESTER

COURSE : MAJOR CORE
PAPER : ANALYTICAL GEOMETRY
TIME : 3 HOURS

MAX. MARKS : 100

SECTION A

Answer All Questions:

10 x 2 = 20

1. Find the centre of the conic $x^2 - 3xy + y^2 + 10x - 10y + 21 = 0$.
2. If the axes are rotated through an angle θ such that $\tan\theta = 2h/(a - b)$, then write the transformed form of the general second degree equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$.
3. If $y = x$ and $3y = -2x$ are a pair of conjugate diameters of an ellipse, then find its eccentricity.
4. Show that the sum of squares of two conjugate diameters is a constant in an ellipse.
5. Write the equation $6x - 3y + 2z - 14 = 0$ of a plane in the normal form.
6. Find the length of perpendicular drawn from the point $(2,1,0)$ to the plane $2x + y + 2z - 17 = 0$.
7. Find the equation of the plane passing through the point $(3,2,-8)$ and is parallel to the plane $3x + y + 2z - 2 = 0$.
8. Prove that the line $\frac{x-1}{2} = \frac{y-3}{3} = \frac{z-4}{-1}$ is parallel to the plane $x - 2y - 4z + 7 = 0$.
9. Find the equation of the tangent plane at the point $(2, -2,1)$ to the sphere $x^2 + y^2 + z^2 - 6x + 2z + 1 = 0$.
10. Define right circular cone.

SECTION B

Answer Any Five Questions:

5 x 8 = 40

11. Find the nature of the curves (i) $4x^2 - 4xy + 4y^2 = 100$ (ii) $x^2 - y^2 = 100$.
12. If CP and CQ are two conjugate diameters of an ellipse, then show that $4(CP^2 - CQ^2) = (SP - S'P)^2 - (SQ - S'Q)^2$.
13. If a straight line cuts a hyperbola in P and Q and its asymptotes in R and S , then prove that $PR = QS$.
14. Show that the points $(0,2,-4)$, $(-1,1,-2)$, $(-2,3,3)$ and $(-3,-2,1)$ are coplanar. Also find the equation of plane on which they lie.
15. If a variable plane which remains at a constant distance $3p$ from the origin cuts the coordinate axes at A,B,C , then show that the locus of the centroid of the triangle ABC is $x^{-2} + y^{-2} + z^{-2} = p^{-2}$.
16. Show that the plane $2x - y - 2z = 16$ touches the sphere $x^2 + y^2 + z^2 - 4x + 2y + 2z - 3 = 0$ and also find the point of contact.
17. Find the equation of the cone with vertex at origin and base curve, the conic in which the surface $ax^2 + by^2 + cz^2 = 1$ is cut by the plane $l_1x + m_1y + n_1z = p$.

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SECTION C

Answer Any Two Questions:

2 x 20= 40

18. a) By changing the origin and the axes, show that the equation $x^2 - 5xy + y^2 + 8x - 20y + 15 = 0$ is reduced to the form $7x^2 - 3y^2 = 2$.
- b) Prove that the tangent to a rectangular hyperbola terminated by its asymptotes is bisected at the point of contact and encloses a triangle of constant area.
19. a) Find the shortest distance between the lines $\frac{x-3}{-1} = \frac{y-4}{2} = \frac{z+2}{1}$; $\frac{x-1}{1} = \frac{y+7}{3} = \frac{z+2}{2}$. Also find the equation of the line of shortest distance.
- b) Find the equation of the plane passing through the line of intersection of the planes $2x + y + 3z - 4 = 0$ and $4x - y + 5z - 7 = 0$ and which is perpendicular to the plane $x + 3y - 4z + 6 = 0$.
20. a) Find the equation of the sphere which passes through the circle $x^2 + y^2 + z^2 - 2x - 4y = 0$, $x + 2y + 3z = 8$ and touches the plane $4x + 3y = 25$.
- b) Find the image of the line $\frac{x-1}{9} = \frac{y-2}{-1} = \frac{z+3}{-3}$ in the plane $3x - 3y + 10z - 26 = 0$.

