**STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086**
**(For candidates admitted from the academic year 2011-12 & thereafter)**

**SUBJECT CODE : 11MT/PC/TO24**

**M. Sc. DEGREE EXAMINATION, APRIL 2015**
**BRANCH I – MATHEMATICS**
**SECOND SEMESTER**

**COURSE : CORE**
**PAPER : TOPOLOGY**
**TIME : 3 HOURS** **MAX. MARKS : 100**

## SECTION – A

**Answer all the questions:** **5×2=10**

1. Define an isometry. When do you say that two metric spaces are isometric?
2. Define a topology and give an example.
3. State Tychonoff' theorem.
4. Sate Urysohn's lemma.
5. Show that the components of a totally disconnected space are it points.

## SECTION – B

**Answer any five questions:** **5×6=30**

6. Let $X$ and $Y$ be metric spaces and $f$ a mapping of $X$into $Y$. Show that $f$ is continuous if and only if $x_n \to x \Rightarrow f(x_n) \to f(x)$.
7. Let $X$ be a second countable space. Show that any open base for $X$ has a countable subclass which is also an open base.
8. Show that a topological space is compact if every basic open cover has a finite sub cover.
9. Show that every compact Hausdorff space is normal.
10. Let $X$ be a locally connected space. If $Y$ is an open subspace of $X$, then show that each component of $Y$ is open in $X$.
11. State and prove Cantor's intersection theorem.
12. Show that any continuous mapping of a compact metric space into a metric space is uniformly continuous.

## SECTION – C

**Answer any three questions:** **3×20=60**

13. (a) Let $A$ be a dense subspace of a metric space $X$ and let $Y$ be a complete metric space. If $f$ is a uniformly continuous mapping of $A$into $Y$, then show that $f$ can be extended uniquely to a uniformly continuous mapping $g$ of $X$ into $Y$.
(b) State and prove Baire's theorem. (14+6)

14. (a) Show that every separable space is second countable.
(b) Let $X$ be any non-empty set, and let $\mathcal{S}$ be an arbitrary class of subsets of $X$. Show that $\mathcal{S}$ can serve as an open subbase for a topology on $X$, in the sense that the class of all unions of finite intersections of sets in $\mathcal{S}$ is a topology. (10+10)

15. (a) State and prove Lebesgue's covering lemma.
(b) State and prove the generalized Heine-Borel theorem. (10+10)

16. State and prove the Urysohn imbedding theorem.

17. (a) Show that a subspace of the real line $\mathbb{R}$ is connected if and only if it is an interval.
(b) Let $X$ be a topological space and $A$ a connected subspace of $X$. Show that $\bar{A}$ is connected.
(c) Define a locally connected space. Given an example of a space which is connected but not locally connected.

(10+5+5)

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