STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2011–12 & thereafter)

SUBJECT CODE: 11MT/PC/PD44

M. Sc. DEGREE EXAMINATION, APRIL 2015
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE	**:**	**CORE**
PAPER	**:**	**PARTIAL DIFFERENTIAL EQUATIONS**
TIME	**:**	**3 HOURS** **MAX. MARKS : 100**

SECTION – A

ANSWER ALL THE QUESTIONS: (5 x 2 = 10)

1. When can you say two partial differential equations $f(x, y, z, p, q) = 0$ and $g(x, y, z, p, q) = 0$ are compatible?
2. Solve $(D^2 - D')z = e^{2x-y}$.
3. State Churchill's problem.
4. State Robin's boundary condition.
5. State some of the fields, where one dimensional equation is used.

SECTION – B

ANSWER ANY FIVE QUESTIONS: (5 x 6 = 30)

6. Solve by Charpits method: $(p^2 + q^2)y = qz$.
7. Find the integral surface of the linear partial differential equation. $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$ containing the straight line $x + y = 0, z = 1$.
8. Solve the equation $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} - \frac{\partial^3 z}{\partial x \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}$.
9. Reduce the partial differential equation $y^2\frac{\partial^2 z}{\partial x^2} - 2xy\frac{\partial^2 z}{\partial x \partial y} + x^2\frac{\partial^2 z}{\partial y^2} = \frac{y^2}{x}\frac{\partial z}{\partial x} + \frac{x^2}{y}\frac{\partial z}{\partial y}$ into canonical form.
10. Derive Poisson equation.
11. State and prove maximum –minimum principle.
12. Obtain the D'Alembert's solution of one dimensional wave equation.

SECTION – C

ANSWER ANY THREE QUESTIONS: (3 x 20 = 60)

13. a) Show that the equations $xp - yq = x, x^2p + q = xz$ are compatible and find their solution.
    b) Use Jacobi's method to solve $p^2x + q^2y = z$.

14. a) Find the solution of the equation $\nabla_1^2 z = e^{-x}\cos y$ which tends to zero as x tends to ∞ and has value $\cos y$ when $x = 0$.
    b) Solve $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = x^2 + xy + y^2$.

15. a) State and solve the interior Dirichlet problem for a circle.

b) Show that in cylindrical coordinates (r,θ,z),Laplace equation has solution of the form $R(r)e^{\pm mz \pm in\theta}$, where $R(r)$ is a solution of the Bessel's equation $\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr} + \left(m^2 - \frac{n^2}{r^2}\right)R = 0$ .If R$\rightarrow 0$ $as\ z \rightarrow \infty$ and is finite when r=0, show that in the usual notation for Bessels functions the appropriate solutions are madeup of terms of the form $J_n(mr)e^{-mz \pm in\theta}$.

16. a) The ends A and B of a rod 10cm in length are kept at temperature $0^{\circ}c$ and $100^{\circ}c$ respectively until the steady state condition prevails. Suddenly the temperature at the end A is increased to $20^{\circ}c$ and the end B is decreased to $60^{\circ}c$.
Find the temperature distribution in rod at time 't'.

b) Find the solution of the one dimensional diffusion equation satisfying the boundary conditions i) T is bounded as $t \rightarrow \infty$ ii) $\frac{\partial T}{\partial x}/_{x=0} = 0$, for all t

iii) $\frac{\partial T}{\partial x}/_{x=a} = 0$, for all t iv) $T(x, 0) = x(a - x), 0 < x < a.$

17. a) By separating the variables, show that one dimensional wave equation $\frac{\partial^2 z}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 z}{\partial t^2}$ has solution of the form $A\exp(\pm inx \pm inct)$, where A and n are constants. Hence show that functions of the form $Z(x,t) = \sum_r (A_r\, cos\frac{r\pi ct}{a} + B_r sin\frac{r\pi ct}{a}) sin\frac{r\pi x}{a}$ where $A_r's$ and $B_r's$ are constants,satisfy the wave equation and the boundary conditions $z(0,t) = 0, z(a,t) = 0$ for all t.

b) Consider Maxwell's equations of electromagnetic theory given by,

$\nabla.\vec{E} = 4\pi\rho,\ \nabla.\vec{H} = 0$, $\nabla \mathrm{x}\vec{E} = \frac{-1}{c}\frac{\partial \vec{H}}{\partial t}$, $\nabla \mathrm{x}\vec{H} = \frac{4\pi i}{c} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$, where $\vec{E}$ is an electric field, ρ is electric charge density, $\vec{H}$ is the magnetic field, i is the current density, c is the velocity of the light . Show that in the absence of charge, $\vec{E}$ and $\vec{H}$ satisfy the wave equation.

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