STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2011-12 & thereafter)

SUBJECT CODE : 11MT/PC/LA24

M. Sc. DEGREE EXAMINATION, APRIL 2015
BRANCH I – MATHEMATICS
SECOND SEMESTER

COURSE : CORE
PAPER : LINEAR ALGEBRA
TIME : 3 HOURS **MAX. MARKS : 100**

Section-A
Answer ALL the questions (5x2=10)

1. Show that similar matrices have the same characteristic polynomial.
2. When do you say that an R-Module M is finitely generated?
3. When a subspace M of V invariant under T becomes cyclic with respect to the nilpotent linear transformation T.
4. Write down the companion matrix of the polynomial $x^4 - 3x^3 + 10x^2 - 5x + 8$.
5. State Principal Axis theorem.

Section-B
Answer any FIVE questions (5x6=30)

6. Let V be a finite-dimensional vector space over the field F and if T is a linear operator on V, then prove that T is triangulable if and only if the minimal polynomial for T is a product of linear polynomials over F.
7. If A and B are submodules of M then prove that
   (i) $A \cap B$ is a submodule of M.
   (ii) $A + B = \{a + b/a \in A, b \in B\}$ is a submodule of M.
8. Prove that there exists a subspace W of V, invariant under T, such that $V=V_1 \oplus W$, where T is a nilpotent linear transformation.
9. Find all possible Jordan canonical forms for a linear operator $T: V \to V$ where minimal polynomial is $x^2(x-1)^2(x+1)^3$.
10. If T in $A_F(V)$ has as minimal polynomial $p(x) = \gamma_0 + \gamma_1 x + \cdots \gamma_{r-1}x^{r-1} + x^r$ and if V as a module is cyclic , then show that there exists a basis of V in which the matrix of T is $C(p(x))$,the companion matrix of $p(x)$.
11. Let V be a finite-dimensional inner product space and if $\mathcal{B} = \{\alpha_1 \ldots \alpha_n\}$ is an orthonormal basis for V and if T is a linear operator on V then prove that
    $A_{kj} = \left(T_{\alpha_j}/\alpha_k\right)$.
12. Prove that on a finite-dimensional inner product space of positive dimension, every self-adjoint operator has a (non-zero) characteristic vector.

Section-C

Answer any THREE questions (3x20=60)

13. State and prove Cayley-Hamilton theorem.
14. Prove that any finitely generated R-module M is the direct sum of a finite number of cyclic submodules where R is a Euclidean ring.
15. (a) Prove that the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 0 \end{bmatrix}$ is nilpotent.Also find its invariants and Jordan form.

    (b) Prove that the matrix $B = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 0 & 0 \end{bmatrix}$ is not similar to $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 0 \end{bmatrix}$
16. Prove that the elements S and T in $A_F(V)$ are similar in $A_F(V)$ if and only if they have the same elementary divisors.
17. (a) Let U be a linear operator on an inner product space V then prove that U is unitary if and only if the adjoint U^* of U exists and $UU^* = U^*U = I$.

    (b) Let V be a finite dimensional inner product space. If f a form on V and T is a linear operator on V then prove that the map f on T is an isomorphism of the space of forms onto $L(V, V)$.

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