STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2011-12 & thereafter)

SUBJECT CODE : 11MT/PC/DG44

M. Sc. DEGREE EXAMINATION, APRIL 2015
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE : CORE
PAPER : DIFFERENTIAL GEOMETRY
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL QUESTIONS : **(5 X 2 = 10)**

1. What is the arc-length of the twisted cubic curve $\Gamma(t) = (t, t^2, t^3)$ starting at $\Gamma(0)$?
2. Define the surface double cone in $\mathbf{R}^3$.
3. Compute the first fundamental form of the sphere
   $s(\theta, \phi) = (cos\theta\ cos\phi, cos\theta\ sin\phi, sin\theta)$
4. What is an umbilic?
5. Define mean curvature of a surface.

SECTION – B

ANSWER ANY FIVE QUESTIONS : **(5 X 6 = 30)**

6. Define reparametrisation. Prove that any reparametrisationof a regular curve is regular.
7. Define torsion of a space curve. Compute the torsion of the circular helix
   $\Gamma(\theta) = (a\ cos\theta, a\ sin\theta, b\ \theta)$.
8. Define tangent space at a point of a surface. If σ is a patch of a surface S containing a point P, (u, v) being coordinates then prove that the tangent space at P is the vector subspace spanned by the vectors σ_u and σ_v
9. Calculate the first fundamental form of the surface $\sigma(u, v) = (u - v, u + v, u^2 + v^2)$
10. If κ_1 and κ_2 are principal curvatures at a point P of a surface patch, prove that κ_1 and κ_2 are real numbers.
11. State and prove Euler's theorem.
12. Prove that the area of a regular surface patch is unchanged by reparametrisation.

SECTION – C

ANSWER ANY THREE QUESTIONS : (3 X 20 = 60)

13. State and prove Serret-Frenet formulae.
14. Prove that the unit sphere and double cone are surfaces.
15. When are two surfaces said to be isometric? Prove that two surfaces S_1 and S_2 are isometric if and only if, for any surface patches σ_1 and $f \circ \sigma_1$ of S_1 and S_2, respectively, have the same first fundamental form.
16. Derive the expression of second fundamental form for a surface. Compute the same for the surface of revolution $\sigma(u, v) = (f(u)\cos v, f(u)\sin v, g(u))$.
17. State and prove Gauss's remarkable theorem.

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