**STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086**
**(For candidates admitted from the academic year 2004–05 & thereafter)**

**SUBJECT CODE: MT/PC/FA44**

**M. Sc. DEGREE EXAMINATION, APRIL 2009**
**BRANCH I – MATHEMATICS**
**FOURTH SEMESTER**

**COURSE : MAJOR CORE**
**PAPER : FUNCTIONAL ANALYSIS**
**TIME : 3 HOURS** **MAX. MARKS : 100**

## SECTION – A

**ANSWER ANY FIVE QUESTIONS: (5 X 8 = 40)**

1. State and prove Holder's and Minkowski's inequalities.
2. Define natural imbedding of $N$ in $N^{**}$ and bring out its properties.
3. State and prove closed Graph theorem.
4. Show that a closed convex set in a Hilbert Space contains a unique vector of smallest norm.
5. If $H$ is a Hilbert Space and $f$ is an arbitrary functional in $N^*$, show that there exists a unique vector $y$ in $H$ such that $f(x) = (x, y)$ for every $x$ in $H$.
6. IF $P$ is a projection in $H$ with range $M$ and null space $N$ prove that $M \perp N$ if and only if $P$ is self adjoint and in this case $N = M^{\perp}$.
7. Prove that an operator $T$ on a Hilbert Space $H$ is unitary if and only if it is an isometric isomorphism of $H$ onto itself.

## SECTION – B

**ANSWER ANY THREE QUESTIONS: (3 X 20 = 60)**

8. a) Let $T$ be a linear transformation from a normed linear space $N$ into a normed linear space $N'$. Prove that the following conditions are equivalent.
   (i) $T$ is continuous.
   (ii) $T$ is continuous at the origin
   (iii) There exists a real number $K > 0$ with the property that $\|T(x)\| \leq K.\|x\|$ for every $x \in N$.
   (iv) If $S$ is the closed unit sphere in $N$ then $T(S)$ is a bounded set in $N'$.

   b) Define $\beta(N, N')$ and prove that it is a Banach Space if $N'$ is a Banach Space.

9. State and prove the Hahn-Banach theorem.

10. a) State and prove the uniform boundedness theorem.
    b) State and prove Bessel's inequality.

11. a) Let $\{e_i\}$ be an orthonormal set in a Hilbert Space $H$. Prove that the following statements are equivalent to one another.

(i) $\{e_i\}$is complete

(ii) $x \perp \{e_i\} \Rightarrow x = 0$

(iii) If $x$ is an arbitrary vector in $H$, prove that $x = \sum (x, e_i) e_i$

(iv) If $x$ is an arbitrary vector in $H$, prove that $\|x\|^2 = \sum |(x, e_i)|^2$.

b) Define adjoint of an operator on a Hilbert Space and prove that the adjoint operation has the following properties.

(i) $(T_1 T_2)^* = T_2 {}^* T_1 {}^*$

(ii) $T^{**} = T$

(iii) $\|T^*\| = \|T\|$

12. State and prove the Spectral theorem.