STELLA MARIS COLLEGE(AUTNOMOUS), CHENNAI-86

(For candidates admitted from 2011-2012)

SUBJECT CODE: 11MT/MC/VL64

B.Sc DEGREE EXAMINATION, APRIL 2014

BRANCH I-MATHEMATICS

SIXTH SEMESTER

COURSE: MAJOR CORE **Time: 3 hours**

PAPER: VECTOR SPACES AND LINEAR TRANSFORMATION **Max marks: 100**

SECTION – A (10 X 2 =20)

ANSWER ANY **TEN** QUESTIONS, EACH QUESTIONS CARRIES 2 MARKS.

1. Give two examples of vector spaces over the field of complex numbers..
2. List out all subspaces of R^2 over R..
3. Write down the condition for any two points in R^2 to be a basis for R^2 over R.
4. Define the dual of a vector space.
5. If S, T $\in Hom(V,W)$ and $v_iS = v_iT$ for all v_i of a basis of V, prove that S = T.
6. Define an inner product space and give an example.
7. Define an orthonormal set of vectors in an inner product space.
8. Show that 0 is an characteristic root of $T \in A(V)$ if and only if T is singular.
9. Define the rank of a linear transformation. If T : V$\rightarrow$ V is an isomorphism of an n-dimensional space V, then what is the rank of T?
10. Define the matrix of a linear transformation $T \in A(V)$ with respect to a basis of V.

SECTION –B (5x8 = 40)

ANSWER ANY FIVE QUESTIONS, EACH QUESTIONS CARRIES 5 MARKS.

11. If $v_1, v_2, ..., v_n$ are in V then prove that either they are linearly independent or some v_k is a linear combination of $v_1, v_2, ..., v_{k-1}$.
12. If S, T are subsets of a vector space V , prove that
    (i). S$\subseteq$T implies L(S) $\subseteq$ L(T).
    (ii). L(L(S)) = L(S).
    (iii). L(S$\cup$T) = L(S) + L(T).
13. State and prove Schwarz inequality in an inner product space.
14. If V is a finite dimensional space over F, then prove that $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not 0.
15. If $\lambda_1, \lambda_2, ..., \lambda_n$ are distinct characteristic roots of $T \in A(V)$ and if $v_1, v_2, ..., v_n$ are characteristic vectors of T belonging to $\lambda_1, \lambda_2, ..., \lambda_n$ respectively, then prove that $v_1, v_2, ..., v_n$ are linearly independent over F.
16. Let U and V be vector spaces with bases B={u_1, u_2, ... , u_n} and B' = {v_1, v_2, ... , v_m}and T: U $\rightarrow$ V a linear transformation. IF u is a vector in U with image T(u), having coordinate vectors a and b relative to these bases, then b = Aa, where A = [T{$u_1)_{B'}$.....T$(u_n)_{B'}$].

17. Let V be a vector space over a field F and **0** be the zero element of V. Prove that (i). $\alpha 0 = 0$, for every $\alpha \in F$ (ii). $ov = 0$, for every v in V (iii). $\alpha(-v) = -(\alpha v)$, for every $\alpha \in F$, $v \in V$ (iv) If $v \neq 0$, then $\alpha v = 0$ implies that $\alpha = 0$.

SECTION –C (2x20 = 40

ANSWER ANY TWO QUESTIONS

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18. (a). Let $V = R^3 = \{(x, y, z) \mid x, y, z \in R\}$, $W_1 = \{(x, y, 0) \mid x, y \in R\}$ and $W_2 = \{(0, 0, z) \mid z \in R\}$. Prove that V is a vector space over R and W_1, W_2 are subspaces of V. Further, prove that V is a direct sum of W_1 and W_2 .
(b). Prove that any two vector spaces of same dimension are isomorphic.
(c). Define annihilator A(W) of a subspace W of a space V and prove that A(W) is a subspace of $\hat{V}$. (7+8+5)

19. (a). If V and W are vector spaces of dimensions of dimensions m and n respectively over F , prove that Hom(V,W) is of dimensions mn over F.
(b). Prove that any finite dimensional inner product space V has an orthonormal space. . (10 + 10)

20. (a). If V is finite dimensional over F and if S, $T \in A(V)$, then prove that
   (i). $r(ST) \leq r(T)$.
   (ii). $r(TS) \leq r(T)$.
   (iii). $R(ST) = r(TS) = r(T)$ for S regular in A(V).
(b). Define Similar matrices and give examples.
(c). Define a diagonalizable matrix. (15+3+2)