STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2011–12)

SUBJECT CODE : 11MT/ME/FM63

B. Sc. DEGREE EXAMINATION, APRIL 2014
BRANCH I – MATHEMATICS
SIXTH SEMESTER

COURSE : MAJOR ELECTIVE
PAPER : FINANCIAL MATHEMATICS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION-A

ANSWER ALL QUESTIONS: **10 X 2 = 20**

1. Define geometric Brownian motion.
2. State the two major flaws in Brownian motion.
3. If you put funds into an account that pays interest at rate r compounded annually, how many years does it take for your funds to double?
4. Define continuously varying interest rates.
5. State the generalized law of one price.
6. Define an American style call option.
7. With usual notation, prove that $\frac{\partial C}{\partial s} = \varphi(\omega)$.
8. Prove that $C(s, t, K, \sigma, r)$ is a decreasing function in K.
9. Define the conditional value at risk in an investment.
10. Write down the formula for rates of return.

SECTION-B

ANSWER ANY FIVE QUESTIONS: **5 X 8 = 40**

11. Show that Geometric Brownian motion is a limit of simpler models.
12. An individual who plans to retire in 20 years has decided to put an amount *A* in the bank at the beginning of each of the next 240 months, after which she will withdraw $1000 at the beginning of each of the following 360 months. Assuming a nominal yearly interest rate of 6% compounded monthly, how large does *A* need to be?
13. State and prove Arbitrage Theorem.

14. Prove that the amount of money needed at time 0 is equal to the expected present value, under the risk-neutral probabilities of the payoff at time 1, by delta hedging arbitrage strategy.
15. Give the limitations of arbitrage pricing and given an example to illustrate the same.
16. State and prove put-call option parity formula.
17. Find the yield curve and the present value function if $r(s) = \frac{r_1}{1+s} + \frac{s\,r_2}{1+s}$.

SECTION-C

ANSWER ANY TWO QUESTIONS: 2 X20 = 40

18. (a) A company needs a certain type of machine for the next five years. They presently own such a machine, which is now worth \$6000 but will lose \$2000 in value in each of the next three years, after which it will be worthless and unusable. The value of its yearly operating cost is \$9000, with this amount expected to increase by \$2000 in each subsequent year that it is used. A new machine can be purchased at the beginning of any year for a fixed cost of \$22000. The lifetime of a new machine is six years, and its value decrease by \$3000 in each of its first two years of use and then by \$4000 in each of the following year. The operating cost of a new machine is \$6000in its first year, with an increase of \$1000 in each subsequent year. If the interest rate is 10%, when should the company purchase a new machine? (10)

    (b) State and prove the Law of One Price and illustrate the same by an example. (10)

19. Derive the Black-Scholes option cost, $C = s\phi(\omega) - Ke^{-rt}\phi(\omega - \sigma\sqrt{t})$.

20. (a) Solve the following Problem.

    Maximize $E[U(W)]$

    Subject to

    $\sum_{i=1}^{n} \omega_i = \omega$; $\omega_i \geq 0,\ i = 1,2,\ldots,n.$

    where U is the investor's utility function. Hence deduce $\omega = 100$, $n = 2$, $r_1 = 0.15$, $v_1 = 0.2$, $r_2 = 0.18$, $v_2 = 0.25$, $\rho = -0.4$, $U(x) = 1 - e^{-0.005x}$. (15)

    (b) Let $C(K,t)$ be the cost of a call option on a specified security that has strike price K and expiration time t. Prove that for fixed expiration time t, $C(K,t)$ is a convex in K. (5)

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