STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted from the academic year 2011-12 & thereafter)

SUBJECT CODE : 11MT/MC/SF44

B. Sc. DEGREE EXAMINATION, APRIL 2014
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE : MAJOR CORE
PAPER : SEQUENCES AND SERIES, FOURIER SERIES
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL THE QUESTIONS: **(10×2=20)**

1. Define one-one function.
2. Prove that the set of integers is countable.
3. Define limit of a sequence.
4. Give an example of a sequence that is oscillating and bounded.
5. Find the limit superior and limit inferior of 1, 2, 3, 1, 2, 3, 1, 2, 3…
6. Define Cauchy sequence.
7. Define alternating series and give an example.
8. Test the convergence of the series 1 + 1 + 1 +…
9. Define even function and give an example.
10. Find the Fourier coefficient a_0 for $f(x) = x - \pi$ in the interval $(-\pi, \pi)$.

SECTION – B

ANSWER ANY FIVE QUESTIONS: **(5×8=40)**

11. If $f: A \to B$ and if $X \subset B, Y \subset B$, Prove that $f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y)$.
12. If $\{s_n\}_{n=1}^{\infty}$ is a sequence of non-negative numbers and if $\lim_{n\to\infty} s_n = L$, Prove that $L \geq 0$.
13. Show that the sequence $\{s_n\}_{n=1}^{\infty}$ where $s_n = \frac{n^2+1}{2n^2+5}, n \in I$ converges to $\frac{1}{2}$.
14. If the sequence $\{s_n\}_{n=1}^{\infty}$ of real numbers is convergent to L, Prove that any subsequence of $\{s_n\}_{n=1}^{\infty}$ is also convergent to L.

15. Prove that the sequence $\{s_n\}_{n=1}^{\infty}$ where $s_n = \left(1+\frac{1}{n}\right)^n$ is convergent.

16. (i) If $0 < x < 1$, Prove that $\sum_{n=1}^{\infty} x^n$ converges to $\frac{1}{1-x}$.

(ii) Prove that the series $\sum_{n=1}^{\infty}\frac{1}{n}$ is divergent.

17. Obtain the half range cosine series for $f(x) = x$ in $(0, \pi)$.

SECTION – C

ANSWER ANY TWO QUESTIONS: **(2×20=40)**

18. a) Prove that $\chi_{A\cup B} = \max(\chi_A, \chi_B)$ for the characteristic function χ.

b) Prove that countable union of countable sets is countable.

c) Prove that every convergent sequence is bounded. (7+7+6)

19. a) If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers with $\lim_{n\to\infty} s_n = L$, and $\lim_{n\to\infty} t_n = M$, then prove that $\lim_{n\to\infty} s_n t_n = LM$.

b) If $\{a_n\}_{n=1}^{\infty}$ is a non increasing sequence of positive numbers such that $\lim_{n\to\infty} a_n = 0$, then prove that the alternating series $\sum_{n=1}^{\infty}(-1)^{n+1}a_n$ is convergent. (10+10)

20. a) Find the half range sine series for $f(x) = \begin{cases} kx, 0 \le x \le \frac{\pi}{2} \\ k(\pi - x), \frac{\pi}{2} \le x \le \pi \end{cases}$.

b) Determine the Fourier series expansion for $f(x) = x$ in the interval $(-\pi, \pi)$.

(10+10)