

B. Sc. DEGREE EXAMINATION, APRIL 2014
BRANCH I – MATHEMATICS
SECOND SEMESTER

COURSE : MAJOR CORE
PAPER : INTEGRAL CALCULUS
TIME : 3 HOURS

MAX. MARKS : 100

SECTION – A

ANSWER ALL THE QUESTIONS:

(10X2=20)

1. Evaluate $\int \log x \, dx$.
2. Evaluate $\int \frac{\cot x}{\log \sin x} \, dx$
3. Evaluate $\int \frac{dx}{4+9x^2}$.
4. Find $\int_0^1 x(1-x)^n \, dx$.
5. Evaluate $\int_0^{\pi/2} \sin^{10} x \, dx$.
6. Evaluate $\int x^2 e^x$ by using Bernoulli's formula for integration by parts.
7. Evaluate $\iint xy \, dx \, dy$ takes over the positive quadrant of the circle $x^2 + y^2 = a^2$.
8. Find the Jacobian of x, y with respect to r, θ if $x = r \cos \theta, y = r \sin \theta$.
9. Define Beta and Gamma function.
10. Prove that $\Gamma(n+1) = n \Gamma(n)$.

SECTION – B

ANSWER ANY FIVE QUESTIONS:

(5X8=40)

11. Evaluate $\int \frac{3x-2}{\sqrt{4x^2-4x-5}} \, dx$.
12. Evaluate $\int_0^{\pi/4} \log(1 + \tan \theta) \, d\theta$.
13. Show that $\int_0^{\pi/2} x^2 \sin x \, dx = \pi - 2$.
14. By changing into polar coordinates evaluate the integral $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} \, dx \, dy$.
15. Evaluate $\int_1^2 \int_1^x xy^2 \, dy \, dx$.
16. Evaluate $\int \int \int xyz \, dx \, dy \, dz$ over the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$ by transforming into spherical coordinates.

17. Evaluate $\int_0^{\infty} e^{-x^2} dx$.

18. Evaluate $\int_0^{\pi/2} \sin^7 \theta \cos^5 \theta d\theta$.

SECTION – C

ANSWER ANY TWO QUESTIONS:

(2X20=40)

19. a) Evaluate $\int (3x - 2)\sqrt{x^2 + x + 1} dx$. (8)

b) If $u_n = \int_0^{\pi/2} x^n \sin x dx$, n is a positive integer. Prove that

$$u_n + n(n - 1)u_{n-2} = n \left(\frac{\pi}{2}\right)^{n-1}. \quad (8)$$

c) Find $\int x \tan^{-1} x dx$. (4)

20. a) Evaluate $\int_0^{\pi/2} \log(\sin x) dx$. (8)

b) Evaluate $\int e^x \frac{x+1}{(x+2)^2} dx$. (4)

c) Change the order of integration in the integral $\int_0^a \int_{x^2/a}^{2a-x} xy dy dx$ and evaluate it. (8)

21. a) Evaluate $\iiint \frac{dx dy dz}{(x+y+z+1)^3}$ taken over the volume bounded by the planes

$$x = 0, y = 0, z = 0, x + y + z = 1. \quad (10)$$

b) Prove the relation between Beta and Gamma function. (10)



