STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2011–12)

SUBJECT CODE : 11MT/MC/GC64

B. Sc. DEGREE EXAMINATION, APRIL 2014
BRANCH I – MATHEMATICS
SIXTH SEMESTER

COURSE : MAJOR CORE
PAPER : GRAPH THEORY AND COMBINETORICS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION-A

ANSWER ALL QUESTIONS: **10 X 2 = 20**

1. Define a bipartite graph and give an example.
2. Prove that $\delta \leq \dfrac{2q}{p} \leq \Delta$.
3. Prove that every cubic graph has an even number of points.
4. Give a graph which is Hamiltonian but not Eulerian.
5. Define centre of a tree.
6. Write the chromatic number of K_8 .
7. Define indegree and outdegree of a vertex.
8. State the pigeonhole principle.
9. Find $P(18; 3, 4, 6)$.
10. Write the ordinary generating function of the sequence $\langle 1,1,1,1,....\rangle$

SECTION-B

ANSWER ANY FIVE QUESTIONS: **5 X 8 = 40**

11. a) Show that in any group of two or more people, there are always two with
   exactly the same number of friends inside the group.
   b) Prove that in any graph G the number of points of odd degree is even. (4 + 4)
12. Prove that a closed walk of odd length contains a cycle.
13. a) Prove that every nontrivial tree G has at least two vertices of degree 1.
   b) Prove that every connected graph has a spanning tree. (4 + 4)
14. Obtain the adjacency and incidence matrix for the following graph.

15. State and prove Euler's theorem for planar graphs.

16. In any connected plane *(p,q)* graph $\left(p \geq 3 \right)$ with *r* faces prove that

$q \geq \dfrac{3r}{2}$ and $q \leq 3p - 6$. Hence prove that K_5 is not planar.

17. Find the Number of (a) 2-digit even numbers, (b) 2-digit odd numbers, (c) 2-digit odd numbers with distinct digits, (d) 2-digit even numbers with distinct digits.

18. Find the coefficient of x^{27} in a) $\left(x^4 + x^5 + x^6 + ... \right)^5$ and b) $\left(x^4 + 2x^5 + 3x^6 + ... \right)^5$

SECTION-C
ANSWER ANY TWO QUESTIONS: **2 X20 = 40**

19. a) Prove that in a graph G any $u - v$ walk contains a $u - v$ path.
   b) Prove that $\Gamma(G) = \Gamma\left(\overline{G} \right)$.
   c) Prove that a graph is bipartite if and only if all its cycles are of even length.

(5 + 5+10)

20. a) Let G be a (p,q) graph. Then prove that following are equivalent.
     i)   G is a tree.
    ii) Every two points of G are joined by a unique path.
   iii) G is connected and $p = q + 1$.
   iv) G is acyclic and $p = q + 1$.
  (b) State and prove five colour theorem.   (10 + 10)

21. a) Find the number of permutations of the digit 1 through 9 in which
    i) none of the blocks 23, 45and678 appears.
   ii) none of the blocks 34, 45and738 appears
  b) Prove the following formula for the Fibonacci numbers:
   $f(n) = C(n,0) + C(n-1,1) + C(n-2,2) + ...$ (10 + 10)

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