STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2011–12)

SUBJECT CODE : 11MT/MC/CA64

B. Sc. DEGREE EXAMINATION, APRIL 2014
BRANCH I – MATHEMATICS
SIXTH SEMESTER

COURSE : MAJOR CORE
PAPER : COMPLEX ANALYSIS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION-A

ANSWER ALL QUESTIONS: **10 X 2 = 20**

1. Represent the function $w = \frac{z}{1+z}$ in the form $f(x, y) = u(x, y) + iv(x, y)$.
2. Is the function $f(z) = \bar{z}$ differentiable? Justify your answer.
3. Define a homothetic transformation.
4. Define the mapping $w = \cos z$.
5. Evaluate, using Cauchy's integral formula, $\frac{1}{2\pi i}\int_C \frac{z^2+5}{z-3}\, dz$ where C is $|z| = 4$.
6. State Morera's theorem.
7. Define removable singularity.
8. Obtain the Taylor's series of the function $f(z) = \frac{z-1}{z+1}$ about $z = 0$.
9. Find the residue of the function $f(z) = \frac{e^z}{z-2}$.
10. State argument theorem.

SECTION-B

ANSWER ANY FIVE QUESTIONS: **5 X 8 = 40**

11. If $f(z) = u(x, y) + iv(x, y)$ is differentiable at $z_0 = x_0 + iy_0$, then prove that $u_x(x_0, y_0) = v_y(x_0, y_0)$ and $u_y(x_0, y_0) = -v_x(x_0, y_0)$.
12. Define cross ratio and prove that any bilinear transformation preserves cross ratio.
13. State and prove Cauchy's inequality and hence prove that a bounded entire function in the complex plane is constant.
14. Let $f(z)$ be analytic everywhere inside of a circle C with center at z_0 and radius R. then prove that $f(z) = \sum_{n=0}^{\infty} \frac{f^{(n)}(z_0)}{n!}(z - z_0)^n$ for all z in $|z - z_0| < R$.

15. State and prove Rouches theorem.

16. Using the method of contour integration, evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$.

17. Evaluate $\int_C \frac{z^2 dz}{(z-2)(z+3)}$ where C is the circle $|z| = 4$.

SECTION-C

ANSWER ANY TWO QUESTIONS: 2 X20 = 40

18. a) Define the harmonic function and prove that the real and imaginary part of an analytic function are harmonic functions.

b) Find the analytic function whose real part is $u(x,y) = 2x - x^3 + 3xy^2$ using Milne-Thomson method.

c) Discuss the transformation $w = z^2$. (6+6+8)

19. a) Obtain the bilinear transformation which maps $-1,0,1$ of the $z-$ plane onto $-1,-i,1$ of the $w-$ plane.

b) Let $f(z)$ be analytic inside and on a simple closed curve C. If z_0 is any point in the interior of C then prove that $f(z_o) = \frac{1}{2\pi i}\int_C \frac{f(z)}{z-z_0}\, dz$.

c) Obtain the Laurent expansion of the function $f(z) = \frac{-1}{(z-1)(z-2)}$ in the region $1 < |z| < 2$. (7+8+5)

20. a) State and prove residue theorem.

b) Using the method of contour integration, prove that $\int_0^{\infty} \frac{dx}{(x^2+1)^2}\, dx = \frac{\pi}{4}$.

c) Prove that the function $f(z)$ is conformal at z_0 if $f(z)$ is analytic at z_o and $f'(z_0) \neq 0$. (6+8+6)

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