

**STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086**  
(For candidates admitted from the academic year 2011-12 & thereafter)

**SUBJECT CODE : 11MT/MC/AG24**

**B. Sc. DEGREE EXAMINATION, APRIL 2014**  
**BRANCH I – MATHEMATICS**  
**SECOND SEMESTER**

**COURSE : MAJOR CORE**  
**PAPER : ANALYTICAL GEOMETRY**  
**TIME : 3 HOURS**

**MAX. MARKS : 100**

**SECTION A**

**Answer All Questions:**

**10 x 2 = 20**

1. Show that the curve given by  $4x^2 - 4xy + 4y^2 = 100$  is an ellipse.
2. Write the condition for the general second degree equation  $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$  to represent a rectangular hyperbola.
3. Find the eccentricity of the ellipse if  $y = x$  and  $3y = -2x$  are a pair of its conjugate diameters.
4. Find the angle between the asymptotes of the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
5. Find the angle between the planes  $2x - y + z = 6$  and  $x + y + 2z = 5$
6. Write the equation of plane in the normal and the intercept forms.
7. Find the equation of the straight line joining the points  $(0,0,0)$  and  $(5, -2,3)$ .
8. Write the condition for the line  $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$  to be parallel to the plane  $ax + by + cz + d = 0$ .
9. Show that the intersection of two spheres represents a plane.
10. Write the condition for the general homogeneous quadric in three variables  $x, y, z$   $F(x, y, z) = ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$  to represent a right circular cone.

**SECTION B**

**Answer Any Five Questions:**

**5 x 8 = 40**

11. Find the centre of the conic given by the general second degree equation  $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$
12. Prove that  $4(CP^2 - CQ^2) = (SP - S'P)^2 - (SQ - S'Q)^2$  where  $CP$  and  $CQ$  are conjugate diameters of an ellipse.

13. The asymptotes of a hyperbola are parallel to  $2x + 3y = 0$  and  $3x - 2y = 0$ . Its centre is at  $(1,2)$  and it passes through the point  $(5,3)$ . Find its equation and its conjugate.
14. Find the equation of plane passing through the points  $(2,1,1)$  and  $(3,2,2)$  and perpendicular to the plane  $x + 2y - 5z = 3$ .
15. Find the equation of the plane which contains two parallel lines  $\frac{x-3}{1} = \frac{y-2}{-4} = \frac{z-1}{5}$  and  $\frac{x-1}{1} = \frac{y+1}{-4} = \frac{z-2}{5}$ .
16. Find the points on the sphere  $x^2 + y^2 + z^2 + 2x - 4z - 4 = 0$  at which the tangent plane is parallel to the plane  $x - 2y - 2z + 1 = 0$ .
17. Find the equation of a right circular cone whose vertex is  $O$ , axis is  $OZ$  and the semi-vertical angle  $\alpha$ .

## SECTION C

Answer Any Two Questions:

2 x 20= 40

18. a. Find the length and positions of the axes of the central conic  $ax^2 + 2hxy + by^2 = 1$ .
- b. Prove that the acute angle between two conjugate diameters of an ellipse is minimum when they are equal.
19. a. Find the equation of the hyperbola conjugate to the hyperbola  $4x^2 + 13xy + 3y^2 + x + 3y - 25 = 0$  and also the equations of the asymptotes.
- b. A variable plane is at a constant distance  $p$  from the origin and meets the axes at  $A, B, C$ . Show that the locus of the centroid of the tetrahedron  $OABC$  is  $x^{-2} + y^{-2} + z^{-2} = 16p^{-2}$
20. a. Prove that the lines  $\frac{x+1}{-3} = \frac{y+10}{8} = \frac{z-1}{2}$  and  $\frac{x+3}{-4} = \frac{y+1}{7} = \frac{z-4}{1}$  are coplanar. Also find the point of intersection.
- b. Find the equation of the sphere which passes through the circle  $x^2 + y^2 + z^2 - 2x - 4y = 0$ ;  $x + 2y + 3z = 8$  and touches the plane  $4x + 3y = 25$ .



