STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2004 – 05 & thereafter)

SUBJECT CODE : MT/MC/RD54

B. Sc. DEGREE EXAMINATION, NOVEMBER 2007
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE : MAJOR – CORE
PAPER : RANDOM VARIABLES AND THEORETICAL DISTRIBUTIONS

TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(10X2=20)**

ANSWER ALL THE QUESTIONS

1. State the axiomatic definition of probability.
2. Two dice are thrown. Find the probability that both the dice show the same number.
3. A, B and C are three mutually exclusive and exhaustive events associated with a random experiment. Find $P(A)$ given that, $P(B) = \frac{3}{2}P(A)$ and $P(C) = \frac{1}{2}P(B)$.
4. Define discrete and continuous distribution functions.
5. Verify whether the following is a distribution function:

$$F(x) = \begin{cases} 0 & if \quad x < -a \\ \frac{1}{2}\left(\frac{x}{a}+1\right) & if \quad -a \le x \le a \\ 1 & if \quad x > a \end{cases}$$

6. State any two properties of a m.g.f.
7. State the uniqueness theorem.
8. Derive the m.g.f of a Poisson distribution.
9. If X is uniformly distributed with mean 1 and variance $\frac{4}{3}$, find P[X<0].
10. Define a normal distributon.

SECTION – B **(5x8=40)**

ANSWER ANY FIVE QUESTIONS

11. State and prove Tchebyshev's inequality.

12. Given the following bivariate probability distribution, find
i) $P[X \le 1, Y = 2]$ ii) $P[X \le 1]$ iii) $P[Y \le 3]$ iv) $P[X < 3, Y \le 4]$.

X	Y					
	1	2	3	4	5	6
0	0	0	$1/32$	$2/32$	$2/32$	$3/32$
1	$1/16$	$1/16$	$1/8$	$1/8$	$1/8$	$1/8$
2	$1/32$	$1/32$	$1/64$	$1/64$	0	$2/64$

13. If $X_1, X_2, ..., X_n$ are random variables then show that
$V\left[\sum_{i=1}^{n} a_i X_i\right] = \sum_{i=1}^{n} a_i^2 V(X_i) + 2\sum_{i=1}^{n}\sum_{j=1}^{n} a_i a_j \operatorname{cov}(X_i, X_j)$. Deduce the result for independent random variables.

14. Let the random variable X have the distribution $P[X = 0] = P[X = 2] = p$;
$P[X = 1] = 1 - 2p$ for $0 \le p \le \frac{1}{2}$. For what value of p is the var X a maximum ?

15. Derive the recurrence relation for moments of the Poisson distribution.

16. Find p for a Binomial variate X if $n = 6$ and $9\ p[X = 4] = p[X = 2]$

17. What are the chief characteristics of normal distribution ?

SECTION – C **(2x20=40)**

ANSWER ANY TWO QUESTIONS

18. a) State and prove Baye's theorem on inverse probability.
b) An urn contains four tickets marked with numbers 112, 121, 211, 222 and one ticket is drawn at random. Let A_i, i = 1, 2, 3 be the event that the ith digit of the number on the ticket is 1. Discuss the independence of the events A_1, A_2 and A_3.

19. a) Show that the correlation coefficient is independent of change of origin and scale.
b) A discrete random variate X takes the values –1, 0, 1 with probability $\frac{1}{8}, \frac{3}{4}, \frac{1}{8}$ respectively. Evaluate $p[|X - \mu| \ge 2\sigma]$ and compare it with the upper bound given by Tchebyshev's inequality..

20. a) Derive the median and mode of normal distribution.
b) The mean yield for one-acre plot is 662 kilos with a standard deviation 32 kilos. Assuming normal distribution, how many one-acre plots in a batch of 1000 plots, would you expect to have yield i) over 700 kilos and ii) below 650 kilos.

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