

B. Sc. DEGREE EXAMINATION, NOVEMBER 2007  
BRANCH I - MATHEMATICS  
FIRST SEMESTER

COURSE : MAJOR – CORE  
PAPER : CLASSICAL ALGEBRA AND MATRIX THEORY  
TIME : 3 HOURS MAX. MARKS : 100

SECTION – A

(10 X 2 = 20)

ANSWER ALL THE QUESTIONS

1. Prove that  $(1+x)^n = 2^n \left\{ 1 - n \frac{1-x}{1+x} + \frac{n(n+1)}{1.2} \left( \frac{1-x}{1+x} \right)^2 \dots \right\}$
2. Show that  $(\log_e 2) - \frac{1}{2!} (\log_e 2)^2 + \frac{1}{3!} (\log_e 2)^3 \dots \infty = \frac{1}{2}$ .
3. Sum the series:  $\frac{1}{(2x-1)} + \frac{1}{3} \cdot \frac{1}{(2x-1)^3} + \frac{1}{5} \cdot \frac{1}{(2x-1)^5} + \dots \infty$
4. The roots of the equation  $8x^3 - 14x^2 + 7x - 1 = 0$  are in Geometric Progression. Find them.
5. If  $\alpha, \beta, \gamma$  are the roots of the equation  $x^3 + px^2 + qx + r = 0$ . Prove that  $(\alpha + \beta)(\beta + \gamma)(\gamma + \alpha) = r - pq$ .
6. Show that  $x^6 + 3x^2 - 5x + 1 = 0$  has atleast four imaginary roots.
7. Solve the equation  $x^3 + 6x + 20 = 0$ , one root being  $1 + 3i$ .
8. Define Skew-Symmetric matrix with an example.
9. Find the rank of the matrix: 
$$\begin{bmatrix} 1 & -1 & 2 \\ 2 & 6 & 3 \\ 3 & 13 & 4 \end{bmatrix}$$
10. Find the eigen values of  $\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$ .

## SECTION – B

(5X8=40)

## ANSWER ANY FIVE QUESTIONS

11. Sum the series to infinity:  $\frac{15}{16} + \frac{15.21}{16.24} + \frac{15.21.27}{16.24.32} + \dots$
12. Sum the series:  $\frac{5}{1!} + \frac{7}{3!} + \frac{9}{5!} + \dots + \infty$
13. Solve the equation:  $6x^5 - x^4 - 43x^3 + 43x^2 + x - 6 = 0$ .
14. If the sum of two roots of the equation  $x^4 + px^3 + qx^2 + rx + s = 0$  equals to the sum of the other two, prove that  $p^3 + 8r = 4pq$ .
15. If  $\alpha, \beta, \gamma$  are the roots of  $x^3 - x - 1 = 0$  show that  $\frac{1+\alpha}{1-\alpha} + \frac{1+\beta}{1-\beta} + \frac{1+\gamma}{1-\gamma} = -7$ .
16. Show that the equation:  $x + y + z = 6$ ,  $x + 2y + 3z = 14$ ,  $x + 4y + 7z = 30$  are consistent and solve them.
17. Obtain the eigen values and eigen vectors of the matrix:  $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

## SECTION – C

(2X20=40)

## ANSWER ANY TWO QUESTIONS

18. a) Sum to infinity of the series:  $\sum_{n=0}^{\infty} \frac{5n+1}{(2n+1)!}$
- b) Show that  $1 + \left(\frac{1}{2} + \frac{1}{3}\right)\frac{1}{4} + \left(\frac{1}{4} + \frac{1}{5}\right)\frac{1}{4^2} + \left(\frac{1}{6} + \frac{1}{7}\right)\frac{1}{4^3} + \dots = \log \sqrt{12}$
19. a) Solve:  $3x^6 + x^5 - 27x^4 + 27x^2 - x - 3 = 0$ .
- b) Solve the equation  $2x^3 - x^2 - 22x - 24 = 0$  given that two of its roots are in the ratio 3:4.
20. a) Diagonalize the matrix  $\begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$
- b) Find the characteristic equation of  $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$  and show that the matrix A satisfies the equation.

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