STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted from the academic year 2004- 05 & thereafter)

SUBJECT CODE : MT/MO/BN24

B. Sc. DEGREE EXAMINATION, APRIL 2007
BRANCH I – MATHEMATICS
SECOND SEMESTER

COURSE : MAJOR OPTIONAL
PAPER : BASICS OF NUMBER THEORY
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL QUESTIONS: **(10 X 2 = 20)**

1. Define the gcd of integers
2. List the prime numbers between 50 and 100.
3. If a/bc and if $(a,b)=1$ prove that a/c.
4. Write down a formula for (a,b) using the prime power factorization of 'a' and 'b'.
5. Use the Euclidean Algorithm to find (360,294)
6. Find $\varphi(960)$.
7. Define the Mobius Function $\mu(n)$. Write down $\mu(7)$.
8. If $ac \equiv bc(\bmod m)$, does it imply $a \equiv b(\bmod m)$. Justify.
9. Find the quadratic residues modulo 7.
10. State the Law of Quadratic Reciprocity.

SECTION – B

ANSWER ANY FIVE QUESTIONS: **(5 X 8 = 40)**

11. If f and g are multiplicative, prove that their Dinichlet product is also multiplicative.
12. State and prove the Euler Format Theorem.
13. If $(562,216)=d$, express 'd' as an integer linear combination of the 2 numbers 562 and 216, using the Euclidean Algorithm.
14. State and prove Lagrange's theorem on Polynomial congruences modulo p.
15. State and prove Wilson's Theorem.
16. Solve: $9x \equiv 12(\bmod 21)$
17. Prove that $\left(\frac{2}{p}\right) = (-1)^{\left(\frac{p^2-1}{8}\right)} = \begin{cases} 1 & if \quad p \equiv \pm 1(\bmod 8) \\ -1 & if \quad p \equiv \pm 3(\bmod 8) \end{cases}$

SECTION – C

ANSWER ANY TWO QUESTIONS: **(2 X 20 = 40)**

18 a) State and prove the Euclidean Algorithm.

b) If $n \geq 1$, prove that $\sum_{d/n} \varphi(d) = n$

c) Let f be multiplicative, prove that f is completely multiplicative if and only if $f^{-1}(n) = \mu(n) f(n)$ for all $n \geq 1$.

(6+6+8)

19. a) Prove that the linear congruence $ax \equiv b(\bmod m)$ where $(a, m) = d$ has solutions iff d/b.

b) Using the Chinese Remainder Theorem, solve the following system of congruences.

x = 1(mod 11)
x = 2(mod 12)
x = 3(mod 13)

(8+12)

20. a) Let p be an odd prime. Then for all n, prove that Legendre Symbol $(n/p) \equiv n^{\left(\frac{p-1}{2}\right)}(\bmod p)$.

b) State and prove Gauss' Lemma. (8+12)