STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2004–05)

SUBJECT CODE : MT/MC/VL64

B. Sc. DEGREE EXAMINATION, APRIL 2007
BRANCH I – MATHEMATICS
SIXTH SEMESTER

COURSE : MAJOR CORE
PAPER : VECTOR SPACES AND LINEAR TRANSFORMATIONS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL QUESTIONS : **(10 X 2 = 20)**

1. Define a vector space and give an example.
2. Determine whether the following vectors are linearly dependent or independent. $(1,3,2)$, $(2,1,0)$, $(0,5,4)$.
3. If S is a subset of a vector space V, show that L(S) is a subspace of V.
4. If $u \in V$, $\alpha \in F$. Prove $\|\alpha u\| = |\alpha| \ \|u\|$.
5. If W is a subspace of a finite dimensional Inner product space, prove that $(W^{\perp})^{\perp} = W$
6. Define a linear transformation.
7. Define a singular element of A(V). Give an example.
8. Find the minimum polynomial of the matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$.
9. If $T \in A(V)$ is regular, then show that $VT = V$.
10. Find the characteristic roots of $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

SECTION – B

ANSWER ANY FIVE QUESTIONS : **(5 X 8 = 40)**

11. If V is a finite-dimensional vector space and if $u_1, u_2, ..., u_m$ span V, then show that a subset of $u_1, u_2, ..., u_m$ forms a basis of V.
12. If A and B are finite-dimensional subspace of a vector space V then show that $\dim(A+B) = \dim(A) + \dim(B) - \dim(A \cap B)$.
13. If V and W are vector spaces over F then show that Hom(V,W) is also a vector space.
14. State and prove Schwarz inequality.
15. If V is finite-dimensional over F. Show that for S, T$\in$ A(V).
    (i) r (ST) $\leq$ r (T)
    (ii) r (TS) $\leq$ r (T)

16. If $\lambda_1, \lambda_2, ..., \lambda_k$ in F are distinct characteristic roots of $T \in A(V)$ and if $v_1, v_2, ..., v_k$ are characteristic vectors of T belonging to $\lambda_1, ..., \lambda_k$ respectively prove that $v_1, v_2, ..., v_k$ are linearly independent over F.

17. If V is n-dimensional over F and if T∈ A(V) has all its characteristic roots in F, prove that T satisfies a polynomial of degree n over F;

SECTION – C

ANSWER ANY TWO QUESTIONS : (2 X 20= 40)

18. a) If $v_1, v_2, ..., v_n$ are in V then show that either they are linearly independent or some v_k is a linear combination of the preceding ones, $v_i, ..., v_{k-1}$.

    b) If $v_1, ..., v_n$ in V have W as linear span and if $v_1, ..., v_k$ are linearly independent, show that we can find a subset of $v_1, v_2, ..., v_n$ of the form $v_1, v_2, ..., v_k$, $v_{i_1}, ..., v_{i_r}$ consisting of linearly independent elements whose linear span is also W.

19. a) If V is a finite-dimensional inner product space, show that V has an orthonormal basis.

    b) If A is an algebra with unit element, over F, then show that A is isomorphic to a sub algebra of A(V) for some vector space V over F.

20. a) If $\lambda \in F$ is a characteristic root of $T \in A(V)$, prove that for any polynomial $q(x) \in F(x)$, q(λ) is a characteristic root of $q(T)$.

    b) If $T \in A(V)$ has all its characteristic roots in F then show that there is a basis of V in which the matrix of T is triangular.

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