STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2004- 05 & thereafter)

SUBJECT CODE : MT/MC/MV24

B. Sc. DEGREE EXAMINATION, APRIL 2007
BRANCH I – MATHEMATICS
SECOND SEMESTER

COURSE : MAJOR CORE
PAPER : MULTIPLE INTEGRALS AND VECTOR ANALYSIS
TIME : 3 HOURS MAX. MARKS : 100

SECTION – A

ANSWER ALL QUESTIONS (10X2=20)

1. Evaluate $\int_0^{\pi/2}\int_0^{\pi/2} \sin(x+y)\ dy\ dx$
2. Change the order of integration $\int_0^a\int_x^a (x^2+y^2)\ dy\ dx$
3. Evaluate $\iint dx\ dy$ over the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.
4. Prove that $\beta(m,n)=\beta(n,m)$
5. If $x=r\cos\theta$, $y=r\sin\theta$ find $\frac{\partial(x,y)}{\partial(r,\theta)}$
6. If $\phi(x,y,z)=x^2y+y^2x+z^2$ find $\nabla\phi$ at $(1,1,1)$.
7. Find the unit normal to the surface $x^2+3y^2+2z^2=6$ at the point $(2,0,1)$.
8. Find div $\bar{r}$.
9. Find the value of 'a' such that $\bar{F}=(axy-z^2)\hat{i}+(x^2+2yz)\hat{j}+(y^2-axz)\hat{k}$, is irrotational.
10. Evaluate $\int \bar{F}.d\bar{r}$ where $\bar{F}=x\hat{i}+y\hat{j}$ and C is the straight line joining $(0,0)$ and $(1,1)$.

SECTION – B

ANSWER ANY FIVE QUESTIONS (5X8=40)

11. Evaluate $\iint_A xy\ dx\ dy$ where A is the domain bounded by the x-axis, ordinate $x=2a$ and the curve $x^2=4ay$.
12. Evaluate $\iint_R r^2\sin\theta\ dr\ d\theta$ over the upper half of the circle $r^2=2a\cos\theta$.
13. Evaluate $\iiint_V 45x^2y\ dv$ where V is the region bounded by the planes $x=0, y=0, z=0, 4x+2y+z=8$.

14. Prove that $\beta(m,n)=\dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

15. Find the equation of the tangent plane and normal to the surface $2xz^2-3xy-4x=7$ at the point $(1,-1,2)$.

16. Prove that $\nabla\times\left(\nabla\times\overline{f}\right)=\nabla\left(\nabla.\overline{f}\right)-\nabla^2\overline{f}$.

17. IF C is a triangle with vertices (0,0), (1,0) and (1,1) find $\oint_C\left(y^2dx+x^2dy\right)$

SECTION – C

ANSWER ANY TWO QUESTIONS **(2X20=40)**

18. a) Prove that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$

b) If $x+y+z=u$, $y+z=uv$, $z=uvw$ prove that $\dfrac{\partial(x,y,z)}{\partial(u,v,w)}=u^2v$.

c) Change the order of integration and hence evaluate $\int_0^b\int_0^{\frac{a}{b}\sqrt{b^2-y^2}} xy\ dx\ dy$

(6+6+8)

19. a) Show that $\overline{F}=\left(2xy+z^3\right)\hat{i}+x^2\hat{j}+3xz^2\hat{k}$ is a conservative vector field. Find a function ϕ such that $\overline{F}=\nabla\phi$. Also find the work done in moving an object in this field from (1,-2,1) to (3,1,4).

b) Find $\nabla^2\log r$. (10+10)

20. a) Evaluate $\iint_S\overline{F}.\hat{n}\ ds$ where $\overline{F}=yz\hat{i}+zx\hat{j}+xy\hat{k}$ and S is the surface of the sphere $x^2+y^2+z^2=1$ which lies in the first octant.

b) Verify Green's theorem for $\int_C(x-2y)dx+xdy$ where C is the circle $x^2+y^2=1$.

(10+10)