STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2004-05 & thereafter)

SUBJECT CODE : MT/MC/LS44

B. Sc. DEGREE EXAMINATION, APRIL 2007
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE : MAJOR CORE
PAPER : LAPLACE TRANSFORMS, SEQUENCES AND SERIES
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL QUESTIONS: **(10 X 2 = 20)**

1. Obtain $L(\sin^2 2t)$.
2. Prove that $L[f'(t)] = sL[f(t)] - f(0)$.
3. Find $L^{-1}\left(\frac{s}{s^2+b^2}\right)$
4. State the least upper bound axiom.
5. Define monotone sequence.
6. Prove that if a sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ converges then $\{s_n\}_{n=1}^{\infty}$ is a Cauchy sequence.
7. Define pointwise convergence of sequence of functions.
8. Define conditional convergence and absolute convergence.
9. State comparison test.
10. Define even and odd function.

SECTION – B

ANSWER ANY FIVE QUESTIONS: **(5 X 8 = 40)**

11. Evaluate $L(t^n)$.
12. Find $L^{-1}\left(\frac{1}{(s^2+a^2)^2}\right)$.
13. Prove that the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}_{n=1}^{\infty}$ is convergent.
14. State and prove nested-interval theorem.
15. State and prove the Cauchy criterion on uniform convergence.
16. If $\{a_n\}_{n=1}^{\infty}$ is a nonincreasing sequence of positive numbers and if $\sum_{n=1}^{\infty} a_n$ converges then prove that $\lim_{n\to\infty} na_n = 0$.
17. Express $f(x) = x(-\pi < x < \pi)$ as a Fourier series with period 2π.
18. Obtain a sine series for f(x) = C in the range 0 to π.

SECTION – C

ANSWER ANY TWO QUESTIONS: (2 X 20= 40)

19. Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{3x}$.

20. a) If $\{s_n\}_{n=1}^{\infty}$ is a sequence of non-negative numbers and if $\lim_{n\to 0} s_n = L$ then prove that $L \geq 0$.
    b) State and prove the fundamental theorem on alternating series.

21. Show that $x^2 = \frac{\pi^2}{3} + \sum_{n=1}^{\infty}(-1)^n \frac{\cos nx}{n^2}$ in the interval $(-\pi \leq x \leq \pi)$.

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