STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2004- 05 & thereafter)

SUBJECT CODE : MT/MC/CT24

B. Sc. DEGREE EXAMINATION, APRIL 2007
BRANCH I – MATHEMATICS
SECOND SEMESTER

COURSE : MAJOR CORE
PAPER : CONICS AND TRIGONOMETRY
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL QUESTIONS : **(10 X 2 = 20)**

1. Prove that every cartesian equation of second degree represents a conic. Find the centre of the conic $5x^2 - 2y^2 + 10x - 4y - 7 = 0$.
2. Show that the sum of the squares of two conjugate semi-diameters of an ellipse is constant.
3. Obtain the equation of the asymptotes of the hyperbola.
4. Find the angle between the asymptotes of the hyperbola.
5. Find the asymptotes of the hyperbola $3x^2 - 5xy - 2y^2 + 17x + y + 14 = 0$.
6. Write down the expansion of $\tan 4\theta$.
7. Expand $\sin^4\theta\cos^2\theta$ in a series of cosines of multiples of θ.
8. If $\sin(A + iB) = x + iy$, prove that $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$.
9. Find $\log(1 - i)$.
10. Write Gregory's series.

SECTION – B

ANSWER ANY FIVE QUESTIONS : **(5 X 8 = 40)**

11. Find the nature of the conic $17x^2 - 12xy + 8y^2 + 46x - 28y + 17 = 0$.
12. P and Q are extremities of two conjugate diameters of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and S is a focus. Prove that $PQ^2 - (SP - SQ)^2 = 2b^2$.
13. Prove that the product of the perpendiculars drawn from any point on a hyperbola to its asymptotes is constant.
14. Express $\frac{\sin 6\theta}{\sin\theta}$ in terms of $\cos\theta$.
15. Prove that $2^6\cos^7\theta = \cos 7\theta + 7\cos 5\theta + 21\cos 3\theta + 35\cos\theta$.
16. Prove that $\tanh^{-1}\left(\frac{x^2 - 1}{x^2 + 1}\right) = \log x \ (x > 0)$.
17. Express $\log\cos(x + iy)$ in the form $A + iB$.

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SECTION – C

ANSWER ANY TWO QUESTIONS : (2 X20 = 40)

18. Find the centre of the conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$.

19. a) The asymptotes of a hyperbola are parallel to $2x + 3y = 0$ and $3x - 2y = 0$. Its centre is at (1,2) and it passes through the point (5,3). Find its equation and its conjugate.
    b) Obtain the equation of a rectangular hyperbola with reference to is asymptotes as axes.

20. a) If x, y, u, v are real numbers such that $u + iv = e^{x+iy}$, prove that $u^2 + v^2 = e^{2x}$ and $v = (\tanh y)u$.
    b) If $\tan\log(x + iy) = a + ib$ where $a^2 + b^2 \neq 1$, prove that $\tan\log(x^2 + y^2) = \dfrac{2a}{1 - a^2 - b^2}$.