STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2004–05)

SUBJECT CODE : MT/MC/CA64

B. Sc. DEGREE EXAMINATION, APRIL 2007
BRANCH I – MATHEMATICS
SIXTH SEMESTER

COURSE : MAJOR CORE
PAPER : COMPLEX ANALYSIS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL QUESTIONS : **(10 X 2 = 20)**

1. Express the function $f(z) = z^3$ in the form $u(x, y) + iv(x, y)$.
2. Find $\lim\limits_{z \to 2i} \left(2x + iy^2\right)^2$.
3. Verify the Cauchy-Riemann equations for the function $f(z) = \sin x \cosh y + i \cos x \sinh y$.
4. Define an analytic function.
5. Prove that the function $u = e^x \cos y$ is harmonic.
6. What is the general form of the bilinear transformation which maps the unit circle $|z| = 1$ onto the unit circle $|w| = 1$?
7. State the maximum modulus theorem.
8. Using Cauchy's integral formula evaluate $\frac{1}{2\pi i} \int_C \frac{z^2 + 6}{z - 2} dz$ where C is $|z| = 3$.
9. State Cauchy's residue theorem.
10. Show that $f(z) = \frac{5}{(z-1)(z-2)^2}$ is a meromorphic function.

SECTION – B

ANSWER ANY FIVE QUESTIONS : **(5 X 8 = 40)**

11. Prove that for the function $f(z) = \begin{cases} \frac{x^3 y(y - ix)}{x^6 + y^2} & if \quad z \neq 0 \\ 0 & if \quad z = 0 \end{cases}$

    the C-R equations are satisfied at $z = 0$ but the function is not differentiable at $z = 0$.

12. Prove that any bilinear transformation preserves cross-ratio.
13. Discuss the mapping $w = \sin z$.

14. Find the analytic function $f(z) = u + iv$ if $u + v = \dfrac{\sin 2x}{\cosh 2y - \cos 2x}$.

15. State and prove Taylor's theorem.

16. Expand $\dfrac{1}{z^2 - 3z + 2}$ in a Laurent's series valid in the region $1 < |z| < 2$.

17. Find the poles of $f(z) = \dfrac{z^2 + 3}{z^3 + 2z^2 + z}$ and determine the residues at the poles.

SECTION – C

ANSWER ANY TWO QUESTIONS : (2 X 20 = 40)

18. a) Derive the Cauchy Riemann equations for a function $f(z)$ which is differentiable at a point z_0.

    b) If $f(z)$ and $\overline{f(z)}$ are analytic in a region D show that $f(z)$ is constant in that region. (12+8)

19. a) Suppose $f(z)$ is analytic inside and on the circle C with centre z_0 and radius r. Then prove that $\left|f^n(z_o)\right| \le \dfrac{n!M}{r^n}$ where M denotes the maximum value of $|f(z)|$ on C. Derive (i) Liouvillie's theorem and (ii) the fundamental theorem of Algebra from the above result.

    b) Evaluate $\int_C \dfrac{z^3 dz}{(2z+i)^3}$ where C is the unit circle. (15+5)

20. a) Using contour integration find the value of $\int_0^{2\pi} \dfrac{d\theta}{2 + \cos\theta}$.

    b) By integrating along a suitable contour show that $\int_0^{\infty} \dfrac{dx}{x^2 + 1} = \dfrac{\pi}{2}$. (10+10)