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# COST ANALYSIS OF A QUEUE SYSTEM WITH IMPATIENT CUSTOMERS

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**ABSTRACT :**
***The paper deals the cost analysis of M/M/1/N queuing system with impatient customers. We assume that the arriving customers balk with some probability and reneging follow negative exponential distribution. The Markov process techniques have been applied to develop the steady state probability. Matrix form solution is given along with some performance measures. A cost model has been developed to find the optimal service rate. To demonstrate the use of the model, different parametric values have been specified and lucid illustration has been presented with the help of table and graphs.***
**Key words:** ***Balking, Reneging, Cost analysis, Exponential distribution, Steady state probability.***



## INTRODUCTION

For any Queuing system, Cost Analysis constitutes a very important aspect of its investigation. A study of queue system seems to be adequate unless comprises of economic analysis. In practical situation, it is nothing but a cost factor which enables to survive a queuing system. Various Researchers discussed the Equilibrium level queue decision model (cost model). Under the analysis of the cost model, they have discussed the two conflicting costs offering the service i.e. the cost of providing the service with the cost of delay in offering the services (waiting time cost) and computed the expected total cost per unit time for the system.

Many researchers find insignificant place for impatient customers with random service. But in modern society & in many real life problems the impatient customers play a meaningful role in a service system and can't be ignored. Such situations involving impatient telephone switch board customers, hospital emergency room which handles critical patient, the inventory system with storage of valuable goods etc.. In this paper, we discussed the performance analysis & the cost model of M/M/1/ N queue system taking the human behavior balking and reneging in account.

Haight (1957, 1959) first of all considered the effect of balking and reneging phenomenon in queuing problem. Ancker & Gafarin (1963) and Abou & Hariri (1992) studied the balking and reneging effect on some queuing problems. Singh T.P (1985) made an attempt to incorporate the reneging concept (reluctant a customer remains in line after joining & waiting) in serial queue network. Man Singh & Umed Singh (1994) studied the steady state behavior for impatient customers. Ashok & Taneja (1983) studied the cost analysis of Multi channel Queue system wherein both the arrival & service intensities are subject to alterations. Neetu Gupta (2009) etal. made an effort of the balking and reneging effect on the performance of a queue system under certain constraints. Recently, Singh T.P & Arti etal. (2014) discussed transient as well as steady state behavior of a serial queue system with reneging.

This work is further, an extended work of Singh T.P. etal. (2014) and it combines the work done by Haight & Neetu Gupta in the sense that both concepts balking and reneging have been included and in addition the cost analysis for the system has been explored with impatient customers through numerically example as well as graphically to demonstrate how the various parameters of the model influence the behavior of the system.

Activating and Deactivating the server may involve power charges equipment or man power charges. The cost can be contributed as a fixed cost which we are not going to consider in our model. When the server is running or busy, fuel and other costs may be charged called running cost or operating cost but these cost do not represent the entire picture of an operation. Consider the case of an air craft repair. The air planes are not productive during they stay in the repair depot and this loss time represents a cost to the owner i.e. we may consider a penalty for delaying the customer in the system, the penalty may be termed as a waiting cost. If the customer balks or renege from the system it is again a loss.

**MODEL DESCRIPTION:**

Model for impatient customers i.e. for balking & reneging stage can be depicted as follow

**ASSUMPTIONS:**

1. Customers arrive at the system one by one in a Poisson stream with mean arrival rate $\lambda$. On arrival a customer either decides to join the queue with probability $q_n$ or balk with probability $1-q_n$ when n customers are ahead (n=1,2,3,4………N-1) where N is maximum number of customers in the system. i.e.

$0 \le q_{n-1} \le q_n < 1 \qquad 1 \le n \le N-1$

$q_n = 0 \qquad n \ge N$

2. After joining queue each customer has to wait a certain time period T for service to start. If the service is not started, he feels irritated and gets impatient and reneges from queue without being served. The time T is a random variable follow exponential distribution and its probability density function is given by, $d(t) = \alpha e^{-\alpha t}$, $t \ge 0$, $\alpha \ge 0$

Here, $\alpha$ is the rate of waiting time T. as the arrival and departure of impatient customers without service are independent, the function of customer's average reneging rate is given by

$$\begin{cases} r(n) = (n-i)\alpha, & i \le n \le N \\ \quad = 0, & n > N \end{cases} \qquad i = 0,1,2,3\ldots\ldots$$

3. Queue discipline is FIFO, once service starts, it is without interruption i.e. it precedes till its completion.

4. We assume service times to be distributed as per exponential distribution whose density function are given by $S(t) = \mu e^{-\mu t}$, $t \ge 0$, $\mu > 0$ Where $\mu$ is service rate

**Steady state differential difference equations:**

Define, $P_n$= probability that there are n customers in the system,

$q_n$ = probability that on arrival a customer decides to join the queue or Balk with probability $1-q_n$ when n customers are ahead.

On Appling the Markov process techniques and elementary probability reasoning the following set of steady state equations has been observed,

$\mu P(1) = \lambda P(0) \qquad$ for n=0

$\lambda q_{n-1} P(n-1) + (\mu + n\alpha) P(n+1) = [\lambda q_n + \mu + (n-1)\alpha)] P(n)$ for n=1,2,3, ….., N-1.

$\lambda q_{N-1} P(N-1) = [\mu + (N-1)\alpha)] P(N)$, for n = N

Now, we discuss the cost analysis of our system For N=4

The following equations are obtained:

$\mu P(1) = \lambda P(0) \qquad$ for n=0 $\qquad$ (1)

$\lambda P(0) + (\mu+\alpha) P(2) = [(\lambda q_1 + \mu)] P(1) \qquad$ for n = 1 $\qquad$ (2)

$\lambda q_1 P(1) + (\mu + 2\alpha) P(3) = [(\lambda q_2 + \mu + \alpha)] P(2) \qquad$ for n = 2 $\qquad$ (3)

$\lambda q_2 P(2) + (\mu + 3\alpha) P(4) = [\lambda q_3 + \mu + 2\alpha)] P(3) \qquad$ for n = 3 $\qquad$ (4)

$\lambda q_3 P(3) = [\mu + 3\alpha)] P(4)$, $\qquad$ for n = 4 $\qquad$ (5)

**Solution methodology:**

Writing the above equations in matrix form:

$$\begin{bmatrix} -\lambda & \mu & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \lambda & -(\lambda q_1 + \mu) & (\mu + \alpha) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \lambda q_1 & -(\lambda q_2 + \mu + \alpha) & (\mu + 2\alpha) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \lambda q_2 & (\lambda q_3 + \mu + 2\alpha) & (\mu + 3\alpha) \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \lambda q_3 & (\mu + 3\alpha) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

By solving as usual method, we get,

$P(0) = k$

$P(1) = \frac{\lambda}{\mu} k$

$P(2) = \lambda^2 \frac{q_1 k}{\mu(\mu+\alpha)}$

$P(3) = \lambda^3 \frac{q_1 q_2 k}{\mu(\mu+\alpha)(\mu+2\alpha)}$

$P(4) = \lambda^4 \frac{q_1 q_2 q_3 k}{\mu(\mu+\alpha)(\mu+2\alpha)(\mu+3\alpha)}$

Applying initial conditions

$\sum_{i=1}^{4} P_i = 1$

i.e. $P(0) + P(1) + P(2) + P(3) + P(4) = 1$

Putting the values we get,

$K = 1 + \frac{\lambda}{\mu} + \lambda^2 \frac{q_1}{\mu(\mu+\alpha)} + \lambda^3 \frac{q_1 q_2}{\mu(\mu+\alpha)(\mu+2\alpha)} + \lambda^4 \frac{q_1 q_2 q_3}{\mu(\mu+\alpha)(\mu+2\alpha)(\mu+3\alpha)}$

Which is the value of P(0)

**Cost Analysis and Performance measures:**

Expected number of customers in queue i.e. waiting customer

$E(N_q) = \sum_{n=1}^{N} (n-1) P(n)$

$E(N) = \sum_{n=1}^{N} n P(n)$

Balking (B.R.) $= \sum_{n=1}^{N} \lambda (1 - q_n) P(n)$

Reneging (R.R.) $= \sum_{n=1}^{N} (n-1) \alpha P(n)$

L.R. = B.R. + R.R.

Where L.R. is cost incurred due to customers loss.

$\mu$= control variable,

Object is to control the service rate to minimize system's total average cost per unit.

$C_1$ = cost per unit time when server is busy.

$C_2$ = cost per unit time when a customer joins in the queue and waits for service.

$C_3$ = cost per unit time when a customer balks or reneges

$F(\mu^*)$= expected functional cost of the system per unit time

Total expected functional cost of the system per unit time

$TF(\mu^*) = C_1 P(B) + C_2 E(N_q) + C_3 L.R.$

P(B) = busy probability of server.

**NUMERICAL ILLUSTRATION**:

Consider maximum number of customer in system N =4,

The probability $q_n = 1/n+1$ & cost elements $C_1=16$, $C_2=10$, $C_3=20$

**Table-1 for α =.1**

| $\lambda \rightarrow$ | .4 | .5 | .6 | .7 | .8 |
|---|---|---|---|---|---|
| $E(N_q)$ | 0.06114 | 0.091609 | 0.125640 | 0.16526 | 0.207158 |
| $E(N)$ | 0.3860 | 0.47839 | 0.56904 | 0.65800 | 0.7452126 |
| L.R. | 0.07542 | 0.114223 | 0.158101 | 0.211769 | 0.27639 |
| $TF(\mu^*)$ | 8.09076 | 10.6553 | 13.3549 | 16.2949 | 19.460916 |
| $\mu^*$ | 0.93458 | 0.88577 | 0.8418 | 0.78834 | 0.72361 |

**Table -2 for λ= .5**

| $\alpha \rightarrow$ | .1 | .2 | .3 | .4 | .5 |
|---|---|---|---|---|---|
| $E(N_q)$ | 0.091609 | 0.0813 | 0.073405 | 0.0667 | 0.0661 |
| $E(N)$ | 0.47839 | 0.46308 | 0.45102 | 0.4412 | 0.43297 |
| L.R. | 0.114223 | 0.118914 | 0.1228 | 0.1260 | 0.131157 |
| $TF(\mu^*)$ | 11.20055 | 11.19128 | 11.19005 | 11.1817 | 11.28 |
| $\mu^*$ | .885777 | 0.8810 | 0.8772 | 0.874 | 0.8688 |

**Analysis of the Table:**

1. We select the fix rate of waiting time α =.1 & change value of arrival rate of customers **λ**, result can be summarized in table-1.
2. We take fix value of **λ**= .5 & change the value of α, the numerical results are shown in table-2.

Table -1 shows that the total expected cost $F(\mu^*)$ increase with increase of **λ**.The expected number of customers in system as well as expected number of waiting customers and average rate of customer loss all increase with increase of **λ**.

Table -2 shows that the total expected cost increases with the increase of α. The expected number of waiting customers in system and average number of customer loss as well as the optimal service rate all decreases with increase in the value of α.

## CONCLUSION

We have discussed the simple queue system with impatient customers and developed the steady state probability equations. The Matrix form of the solution has been derived. We formulate a cost model to determine the optimal service rate and total expected cost of the system per unit time. Although this function is too complicated to derive the explicit expression for optimal service rate, even than we have made an attempt to evaluate numerically the performance measures & the optimal service rate for the system. We have also presented how the various parameters of the model influence the behavior of the system.

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