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FUZZY MODELING OF CORTISOL SECRETION OF JOB STRAIN DUE TO STRESS USING EXTENDED HAUSDROFF DISTANCES FOR INTUITIONISTIC FUZZY SETS

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ABSTRACT :
In this paper, Fuzzy model of extended hausdroff distance for intuitonistic fuzzy sets and interval-valued fuzzy sets based on the hausdroff metric (four inputs–one output and two inputs–one output) were developed to test the hypothesis that high job demands and low job control (job strain) are associated with elevated free cortisol levels early in the working day and with reduced variability across the day and to evaluate the contribution of anger expression to this pattern. The quality of the model was determined by comparing predicted and actual fuzzy classification and defuzzification of the predicted outputs to get crisp values for correlating estimates with published values. A modified form of the Hamming distance and Euclidian distance measure is proposed to compare predicted and actual fuzzy classification. An entropy measure is used to describe the ambiguity associated with the predicted fuzzy outputs. The two inputs (high and low) predicted over 40% of the test data within one-half of a fuzzy class of the published data. Comparison of the model (men and women) shows that the hausdroff hamming distances exhibited less entropy than the hausdroff Euclidian distance.

Key words: ***job strain, cortisol, anger, work stress, teaching, Intuitionistic fuzzy set, Mamdani fuzzy modeling, Hamming distance, Hausdroff Hamming distance, Hausdroff Euclidean distance, Extended Hausdroff distance***
2000 Mathematics Subject Classification: Primary 90B22 Secondary 90B05; 60K30

1. INTRODUCTION

Fuzzy set was proposed by Zadeh in 1965 as a frame work to encounter uncertainty, vagueness and partial truth. It represents a degree of membership for each member of the universe of discourse to a subset of it. Intuitionistic fuzzy set was proposed by Attanassov, [2] in 1986 which looks more accurate to uncertainty quantification and provides the opportunity to precisely model the problem based on the existing knowledge and observations. The Intuitionistic fuzzy set theory has been applied in different areas.

Fuzzy model has proved valuable in understanding the work characteristics associated with coronary heart disease risk, hypertension, mental health, quality of life, and other outcomes. This model proposes that people working in highly demanding jobs who also have low control and limited opportunities to use skills will experience high job strain. The HPA axis is one of the principal pathways activated as part of the physiological stress response. Using the concept of an intuitionistic fuzzy set that makes it possible to express many new aspects of imperfect information. For instance, in many cases information obtained cannot be classified due to lack of knowledge, discriminating power of measuring tools, etc. In such a case the use of a degree of membership and non-membership can be an adequate knowledge representation solution.

The hausdroff distances play an important role in practical application, notably in image matching, image analysis, motion tracking, visual navigation of robots, computer-assisted surgery and so on. We therefore, obtained measures of salivary cortisol throughout the working day, evening, assessed differences between day and evening as well as early morning levels. We hypothesized that job strain would be associated with elevated cortisol early in the morning together with heightened cortisol later in the day. Such a pattern might lead to reduced variability in cortisol output over the working day. An additional aim of this study was to investigate possible interactions between Job **Strain and Anger Expression.**

2. MAMDANI-TYPE FUZZY MODELING

This paper presents a Mamdani fuzzy modeling scheme where rules are derived from multiple knowledge sources such as previously published databases and models, existing literature, intuition and solicitation of expert opinion to verify the gathered information. The output or consequence of a Mamdani-type model is represented by a fuzzy set. To assess model performance, a crisp estimate of the consequence is usually made by defuzzification methods such as the centroid, weighted average, maximum membership principle and mean membership principle [3]. Depending on the shape of the output fuzzy set, defuzzification methods do not effectively characterize the output with the corresponding ambiguity associated with the prediction. An alternative strategy could be implemented such that the actual values of the output infer an ordinal set representing a three point fuzzy classification (low and high) that could be compared to the actual fuzzy classification using distance measures. In addition, the ambiguity associated with the predicted fuzzy sets can be quantified by calculating entropy [4].

A stochastic model for psychological effect of **Compassion & Anger** was explored by [11]. The purpose of this study was to develop generalized rule based fuzzy models from multiple knowledge sources to test the hypothesis that high job demands and low job control (job strain) are associated with elevated free cortisol levels early in the working day and with reduced variability across the day and to evaluate the contribution of **Anger Expression** to this pattern and subsequently test its performance by comparing defuzzified outputs to actual values from test data and comparing predicted and actual fuzzy classifications. The overall approach followed in this study is illustrated in Figure 1. The process begins with knowledge acquisition, continues to model building and then finally testing the model performance. In the context of fuzzy modeling, the proposed approach of converting the predicted fuzzy output and the actual crisp value into fuzzy classification sets is not well defined in literature.

Each row of membership functions constitutes an IF– THEN rule, also defined by the user. Depending on the values used, the input membership functions are activated to a certain degree. The contributed output from each rule reflects this degree of activation. The final output is a fuzzy set created by the superposition of individual rule actions (Figure 1).

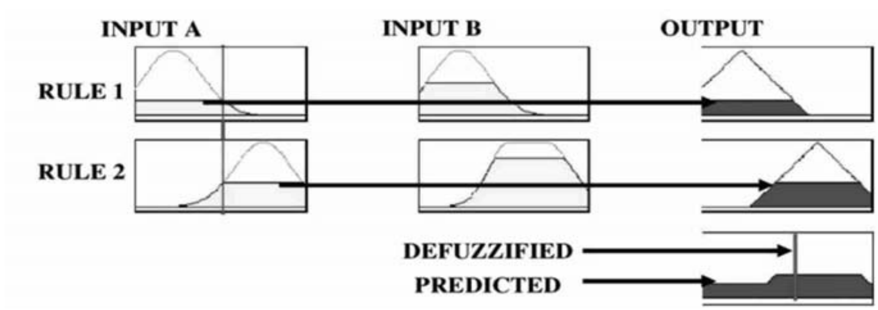

Figure1: Examples of a Mamdani type fuzzy inference system

2.1 DEFUZZIFICATION METHODS

The fuzzy output is obtained from aggregating the outputs from the firing of the rules. Subsequent defuzzification methods on the fuzzy output produce a crisp value. Two common techniques for defuzzification are the maxima methods and area-based methods, which are briefly explained. Several such methods are explained by Ross (1995).

3. DISTANCE MEASURES BETWEEN FUZZY SETS

For two fuzzy sets A and B in the same universe, the Hamming distance (HD) [5] is an ordinal measure of dissimilarity and is defined as:

$$\mathrm{HD(A,B)} = \sum_{i=1}^{n} \left| \mu_A(x_i) - \mu_B(x_i) \right|$$

where n is the number of points that define the fuzzy sets A and B, $\mu_A(x_i)$ the membership of point x_i in A and $\mu_B(x_i)$ is the membership of point x_i in B. The Hamming distance is smaller for fuzzy sets that are more alike than those that are less similar.

3.1 ENTROPY OF A FUZZY SET

Entropy is a measure of fuzziness associated with a fuzzy set. The degree of fuzziness can be described in terms of a lack of distinction between a fuzzy set and its complement. For a fuzzy set A, entropy [6] is calculated as:

$$\mathrm{E(A)} = 1 - \frac{1}{n} \sum_{i=1}^{n} \left| 2\mu_A(x_i) - 1 \right| \qquad \text{.................(1)}$$

where n is the number of points that define A, and $\mu_A(x_i)$ is the membership of point x_i in A. In this study, the concept of entropy was used to quantify the ambiguity associated with the predicted fuzzy outputs. In the absence of actual values, entropy values are essentially a measure of confidence in outputs predicted by a fuzzy model.

3.2 PROPOSED DISTANCE MEASURE

As indicated in the theory section, a modified form of the Hamming distance is proposed which enables better distinction between different levels of classification (see Table1 and 2).

The proposed distance measure D(A, P) is defined as:

$$\mathrm{D(A,B)} = \frac{1}{4}\left(\sum_{i=1}^{n} \left| \mu_A(x_i) - \mu_B(x_i) \right| + \sum_{i,k=1(i \neq k)}^{n} (2(2\left| i-k \right| - 1)\mu_A(x_i)\mu_P(x_k)\right) \quad \text{......(2)}$$

where A is the actual fuzzy classification, P the predicted fuzzy classification, n the number of classes that define A and P, $\mu_A(x_i)$ is the membership of point x_i in A and $\mu_P(x_k)$ is the membership of point x_k in P.

3.3 COMPARING FUZZY CLASSIFICATIONS

The two output membership functions created in both models are categorized as low and high. The actual value from the test data was evaluated using the parameters of these membership functions to produce a fuzzy set represented by two points (high and low). This fuzzy set represents the degree of belongingness (μ) to each of the two categories (low and high). The predicted output from the Mamdani model is a fuzzy set represented by the given points. Based on the relative contributions from each output membership function, the predicted fuzzy set of given points was reduced to a fuzzy set of three points. The relative contributions from each output membership function were estimated by integrating the predicted fuzzy set over the range of the membership function. Equations (3) were used to develop the predicted fuzzy classification:

$$\mu_P(x) = \frac{\int f(x).\mu_i(x)dx}{\int \mu_i(x)dx} \quad \ldots\ldots\ldots\ldots(3)$$

For each test case, an actual fuzzy classification and a predicted fuzzy classification were obtained. The modified Hamming distance measure (3) was used to determine the similarity between the two fuzzy sets. Apart from a comparison to actual values, the ambiguity associated with each predicted value was quantified using an entropy measure (1) as defined in the theory section.

3.4 DEFUZZIFYING THE PREDICTED OUTPUT

The centroid method was used to defuzzify the output of the Mamdani models. The crisp predictions were compared to the actual values from the test data and entropy value was calculated. This is a common form of comparison utilized for most modeling strategies. However, defuzzifying the output results in a loss of information regarding the ambiguity of the prediction. In the absence of actual values, the confidence in the prediction can be determined based on the degree of ambiguity.

3.5 INTUITIONISTIC FUZZY SETS

Intuitionistic fuzzy set was introduced first time by Atanassov, which is a generalization of an ordinary Zadeh fuzzy set. Let X be a fixed set. An intuitionistic fuzzy set A in X is an object having the form $A = \{(x, \mu_A(x), v_A(x))|\ x \in X\}$ where the functions $\mu_A(x), v_A(x) : X \rightarrow [0,1]$ are the degree of membership and the degree of non-membership of the element $x \in X$ to A , respectively; moreover, $0 \leq \mu_A(x) + v_A(x) \leq 1$ must hold.

Obviously, each fuzzy set may be represented by the following intuitionistic fuzzy set

$$A = \{(x, \mu_A(x), 1 - \mu_A(x)|\ x \in X\}$$

3.6 DISTANCE BETWEEN INTUITIONISTIC FUZZY SET

In Szmidt and Kacprzyk [7], [8], it is shown why in the calculation of distances between the intuitionistic fuzzy sets one should use all three terms describing them. Let A and B be two intuitionistic fuzzy set in $= \{x_1, x_2, \ldots, x_n\}$. Then the distance between A and B while using the three term representation (Szmidt and Kacprzyk) may be as follows.

The Hamming distance:

$$d_{IFS}(A,B) = \frac{1}{2}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)| + |v_A(x_i) - v_B(x_i)|$$

The Euclidean distance:

$$e_{IFS}(A,B) = \sqrt{\frac{1}{2}\sum_{i=1}^{n}((\mu_A(x_i) - \mu_B(x_i))^2 + (v_A(x_i) - v_B(x_i))^2)}$$

The normalized Hamming distance:

$$l'_{IFS}(A,B) = \frac{1}{2n}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)| + |v_A(x_i) - v_B(x_i)|)$$

The normalized Euclidean distance:

$$q'_{IFS}(A,B) = \sqrt{\frac{1}{2n}\sum_{i=1}^{n}((\mu_A(x_i) - \mu_B(x_i))^2 + (v_A(x_i) - v_B(x_i))^2}$$

3.7 THE HAUSDROFF DISTANCE

Given two intervals $U = [u_1, u_2]$ and $V = [v_1, v_2]$ of Intuitionistic fuzzy set, the hausdroff metric is defined [9], $d_H(U,V) = \max\{|u_1 - v_1|, |u_2 - v_2|\}$. The hausdroff metric applied to two intuitionistic fuzzy sets, $A(x) = [\mu_A(x), 1 - v_A(x)]$ and $B(x) = [\mu_B(x), 1 - v_B(x)]$, given the following:

$$d_H(A(x), B(x)) = \max\{|\mu_A(x_i) - \mu_B(x_i)|, |v_A(x_i) - v_B(x_i)|\}$$

The following two term representation hausdroff distances between intuitionistic fuzzy sets have been proposed [9]:

The Hamming distance:

$$d_H(A,B) = \sum_{i=1}^{n} \max\ \{|\mu_A(x_i) - \mu_B(x_i)|, |v_A(x_i) - v_B(x_i)|\}$$

The normalized Hamming distance:

$$l_H(A,B) = \frac{1}{n}\sum_{i=1}^{n} \max\ \{|\mu_A(x_i) - \mu_B(x_i)|, |v_A(x_i) - v_B(x_i)|\}$$

The Euclidean distance:

$$e_H(A,B) = \sqrt{\sum_{i=1}^{n} max((\mu_A(x_i) - \mu_B(x_i))^2, (v_A(x_i) - v_B(x_i))^2)}$$

The normalized Euclidean distance:

$$q_H(A,B) = \sqrt{\frac{1}{n}\sum_{i=1}^{n} max((\mu_A(x_i) - \mu_B(x_i))^2, (v_A(x_i) - v_B(x_i))^2)}$$

The Extended haudroff - normalised Hamming distance:

$$\boldsymbol{l_{EH}(s(p_i), d_k) = \frac{1}{5}\sum_{j=1}^{5} max\{|\mu_j(p_i) - \mu_j(d_k)|\}}$$

In comparing an actual fuzzy set to the predicted fuzzy set, a small Hamming distance is ideal. In our study, the model-testing phase involved comparison of predicted and actual fuzzy classifications (low and high). From the results in Table 1, the proposed distance measure is better than the Hamming distance at distinguishing between different levels of classification. In cases e and f, the Hamming distance(HD) gave the same value for different predicted fuzzy classifications[10]. The extended Hausdroff distance gave different values that effectively distinguish between these cases.

4. EXAMPLE

Data were collected at the 12-month follow-up phase of a study of job strain and cardiovascular risk, details of which have been published previously [10]. Participants in the original sample were 162 junior and high school teachers, selected on the basis of scores on a work stress measure (37) as having high (28 men and 52 women) or low (32 men and 50 women) job strain scores. Eighty-five (52.5%) were classroom teachers, and 77 (47.5%) had additional administrative roles. One hundred thirty-seven teachers took part in the 12-month phase (84.6%), which consisted of ambulatory blood pressure monitoring and a psychiatric interview (to be reported elsewhere) in addition to cortisol measurements. Of the 25 who did not participate at 12 months, 10 had left teaching or retired, 7 were seriously ill or pregnant, 1 experienced equipment failure, and 7 did not respond to our invitation. Comparisons between the 137 participants and 25 who dropped out of the study revealed no significant differences in gender, job strain scores, age, grade of employment, or scores on negative affect or anger expression. An additional 15 of the 137 individuals refused to sample saliva during the working day, mainly because they envisaged that data collection might be embarrassing or inconvenient at school. Statistical comparisons of these individuals with the remainder again identified no differences on demographic or psychological variables.

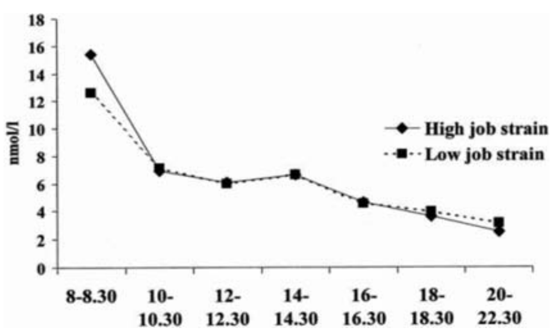

Figure 2. Mean concentration of saliva free cortisol in high and low job strain groups across the day and evening.

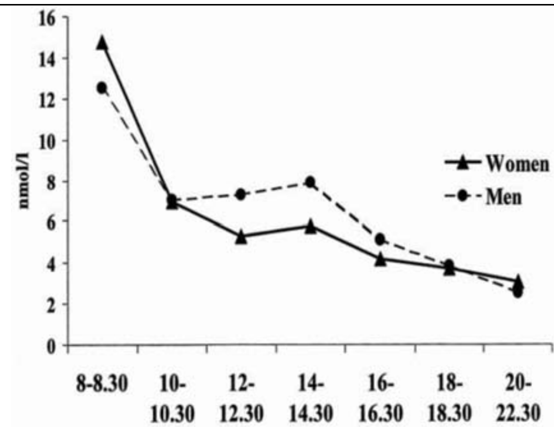

Figure 3. Mean concentration of saliva free cortisol in men and women across the day and evening.

Fuzzy function of the given figure 2 and 3 is defined as

$$f(x)=\begin{cases}-5x+1.5, x\in[0,0.2]\\ 0.5, x\in[0.2,0.4]\\ -x+0.9, x\in[0.4,0.7]\end{cases}$$

$$A_1(x)=\begin{cases}6.67x, x\in[0,0.15]\\ 1, x\in[0.15,0.25]\\ -6.67x+2.67, x\in[0.25,0.4]\\ 0, otherwise\end{cases}$$

$$A_2(x)=\begin{cases}5x-1, x\in[0.2,0.4]\\ 1, x\in[0.4,0.5]\\ -5x+3.5, x\in[0.5,0.7]\\ 0, otherwise\end{cases}$$

Corresponding Fuzzy diagram are given in figure 4.

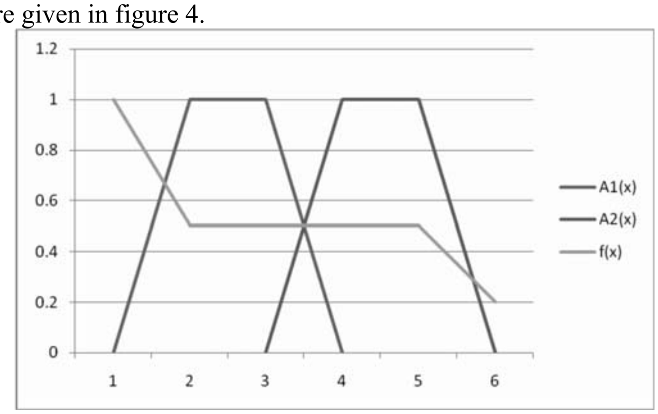

Figure 4. Fuzzy Mean concentration of saliva free cortisol in high and low job strain groups across the day and evening and Fuzzy Mean concentration of saliva free cortisol in men and women across the day and evening.

Saliva sampling was conducted on a working day at schools. Participants were asked to take eight saliva samples at 2-hour intervals, and a 30-minute time window was allowed for each sample. Participants were asked to not consume any caffeine, citrus drinks, or food for at least 60 minutes before the saliva sample was taken. The schedule sampling sequence was therefore 8:00 to 8:30, 10:00 to 10:30, 12:00 to

12:30, 14:00 to 14:30, 16:00 to 16:30, 18:00 to 18:30, 20:00 to 20:30, and 22:00 to 22:30 hours. The first sample of the day was always obtained at schools after explanation of the procedure by the investigators. Saliva samples were collected in Salivettes, which were stored at −30°C until analysis. After defrosting, samples were spun at 3000 rpm for 5 minutes, and 100 µl of supernatant was used for duplicate analysis involving a time-resolved immunoassay with fluorescence detection.

Case		Actual fuzzy classification		Predicted fuzzy classification		HD	Predicted Distance
	Time	High $\mu_A(x_i)$	Low $\mu_A(x_i)$	High $\mu_B(x_i)$	Low $\mu_B(x_i)$		
a	8 – 8.30	(1, 0)	(0.83, 0.17)	(0.9, 0.1)	(0.8, 0.2)	0.13	0.43
b	10 – 10.30	(0.45, 0.55)	(0.45, 0.55)	(0.8, 0.2)	(0.8, 0.2)	0.4	0.34
c	12 – 12.30	(0.39, 0.61)	(0.39, 0.61)	(0.5, 0.5)	(0.5, 0.5)	0.16	0.13
d	14 – 14.30	(0.44, 0.54)	(0.44, 0.54)	(0.4, 0.6)	(0.4, 0.6)	0.16	0.11
e	16 – 16.30	(0.31, 0.69)	(0.31, 0.69)	(0.5, 0.5)	(0.5, 0.5)	0.4	0.18
f	18 – 18.30	(0.24, 0.76)	(0.27, 0.73)	(0.4, 0.6)	(0.5, 0.5)	0.4	0.16
g	20 – 22.30	(0.17, 0.83)	(0.21, 0.79)	(0.4, 0.6)	(0.5, 0.5)	0.51	0.18
Entropy Value		(0.57, 0.43)	(0.64, 0.36)	(0.64, 0.36)	(0.8, 0.2)		

Various Distance Between Intuitionistic fuzzy set	Women	Men
Hamming Distance	0.54	0.55
Euclidean Distance	0.39	0.49
Normalized Hamming Distance	0.15	0.16
Normalized Euclidean Distance	0.15	0.19
Hausdorff Hamming Distance	**0.53**	**0.54**
Hausdorff Euclidean Distance	**0.39**	**0.49**
Hausdorff Normalized Hamming Distance	**0.15**	**0.16**
Hausdorff Normalized Euclidean Distance	**0.15**	**0.19**

Table 1: Comparison of the various distances of Fuzzy Mean concentration of saliva free cortisol in high and low job strain groups across the day and evening

Case		Actual fuzzy classification		Predicted fuzzy classification		HD	Predicted Distance
	Time	Women $\mu_A(x_i)$	Men $\mu_A(x_i)$	Women $\mu_B(x_i)$	Men $\mu_B(x_i)$		
a	8 – 8.30	(1, 0)	(0.83, 0.17)	(0.9, 0.1)	(0.8, 0.2)	0.13	0.43
b	10 – 10.30	(0.6, 0.4)	(0.6, 0.4)	(0.8, 0.2)	(0.8, 0.2)	0.7	0.35
c	12 – 12.30	(0.34, 0.66)	(0.5, 0.5)	(0.5, 0.5)	(0.5, 0.5)	0.22	0.15
d	14 – 14.30	(0.37, 0.63)	(0.53, 0.47)	(0.4, 0.6)	(0.4, 0.6)	0.08	0.11
e	16 – 16.30	(0.27, 0.73)	(0.33, 0.67)	(0.5, 0.5)	(0.5, 0.5)	0.38	0.17
f	18 – 18.30	(0.25, 0.75)	(0.25, 0.75)	(0.4, 0.6)	(0.5, 0.5)	0.39	0.16
g	20 – 22.30	(0.2, 0.8)	(0.19, 0.81)	(0.4, 0.6)	(0.5, 0.5)	0.52	0.17
Entropy Value		(0.52, 0.48)	(0.66, 0.36)	(0.71, 0.29)	(0.8, 0.2)		

Various Distance Between Intuitionistic fuzzy set	Women	Men
Hamming Distance	0.59	0.62
Euclidean Distance	0.51	0.56
Normalized Hamming Distance	0.17	0.18
Normalized Euclidean Distance	0.19	0.21
Hausdorff Hamming Distance	**0.58**	**0.61**
Hausdorff Euclidean Distance	**0.51**	**0.56**
Hausdorff Normalized Hamming Distance	**0.18**	**0.17**
Hausdorff Normalized Euclidean Distance	**0.19**	**0.21**

Table 2: Comparison of the Hamming various distances of Fuzzy Mean concentration of saliva free cortisol in men and women across the day and evening.

Extended Hausdroff Distance

High and Low	0.152 and 0.154
Women and Men	0.16 and 0.168

CONCLUSION

There were significant differences between groups in job strain and in its components job demands, job control, and skill utilization. The high job strain group reported greater demands, lower control, and less skill utilization than the low job strain group as inputs. Negative affect was significantly higher among high job strain individuals, and anger-in scores were also greater. There were no differences in anger-out ratings between groups. Using multiple knowledge sources, membership functions and rules were developed to provide generalized models not optimized for a specific data set. Apart from correlation estimates of actual and defuzzified predictions, an alternative analysis was performed involving comparison of actual and predicted fuzzy classifications. Various distances measure were used to compare actual and fuzzy classifications. The extended hausdroff distance often used to compare distances between fuzzy sets.

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