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CONTRIBUTION OF RAMANUJAN IN MODERN MATHEMATICS

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ABSTRACT :
In this article, the contribution of Great Indian Mathematician Srinivasa Ramanujan in the development of modern mathematics is described. Starting with the early life, his journey from a small village in Kumbakonam to the Fellow of the Royal Society is described in detail. Particular attention is given to his extraordinary research work which formed a strong foundation in the development of analysis, number theory, infinite series and continued fractions.

INTRODUCTION

Srinivasa Ramanujan (22 December 1887 – 26 April 1920) was an Indian mathematician who, with almost no formal training in pure mathematics, made extraordinary contributions to mathematical analysis, number theory, infinite series, and continued fractions. Living in India with no access to the larger mathematical community, Ramanujan developed his own mathematical research in isolation. As a result, he rediscovered known theorems in addition to producing new ones. Ramanujan was said to be a natural genius by the English mathematician G. H. Hardy, in the same league as mathematicians such as Euler and Gauss. His introduction to formal mathematics began at age 10 when he was given books on advanced trigonometry written by S. L. Loney that he mastered by the age of 12. He even discovered theorems of his own, and re-discovered Euler's identity independently. Ramanujan received a scholarship to study at Government College in Kumbakonam, which was later withdrawn when he failed in his non-mathematical coursework. He joined another college to pursue independent mathematical research, working as a clerk in the Accountant-General's office at the Madras Port Trust Office to support himself. In 1912–1913, he sent samples of his theorems to three academics at the University of Cambridge. G. H. Hardy, recognizing the brilliance of his work, invited Ramanujan to visit and work with him at Cambridge. He became a Fellow of the Royal Society and a Fellow of Trinity College, Cambridge. Ramanujan died of illness, malnutrition, and possibly liver infection in 1920 at the age of 32.

During his short lifetime, Ramanujan independently compiled nearly 3900 results and nearly all his claims have now been proven correct. He stated results that were both original and highly unconventional, such as the Ramanujan prime and the Ramanujan theta function, which have inspired and inspiring a vast amount of further research. In December 2011, in recognition of his contribution to mathematics, the Government of India declared that Ramanujan's birthday (22 December) should be celebrated every year as **National Mathematics Day**, and also declared 2012 the **National Mathematics Year.**

THE LIFE OF SRINIVASA RAMANUJAN

EARLY LIFE

Ramanujan was born in Erode, Madras Presidency (now Tamil Nadu), at the residence of his maternal grandparents. His father, K. SrinivasaIyengar, worked as a clerk in a sari shop and his mother was a housewife and also sang at a local temple. In December 1889, Ramanujan had smallpox and recovered. He moved with his mother to her parents' house in Kanchipuram, near Madras (now Chennai). On 1 October 1892, Ramanujan was formally enrolled at the local school. Just before the age of 10, in November 1897, he passed his primary examinations in English,Tamil, Geography and Arithmetic. With his scores, he stood first in the district and entered Town Higher Secondary School where he encountered formal mathematics for the first time.

In 1903 when he was 16, Ramanujan obtained from a friend a library-loaned copy of the book titled *A Synopsis of Elementary Results in Pure and Applied Mathematics* by G. S. Carrandcontaining 5000 theorems. The book is generally acknowledged as a key element in awakening the genius of Ramanujan. He had independently developed and investigated the Bernoulli numbers and had calculated the Euler–Mascheroni constant up to 15 decimal places. When he graduated from Town Higher Secondary School in 1904, Ramanujan was awarded the K. RanganathaRao prize for Mathematics by the school's headmaster by introducing him as an outstanding student who deserved scores higher than the maximum possible marks. He received a scholarship to study at Government Arts College, Kumbakonam, However, Ramanujan was so intent on studying mathematics that he could not focus on any other subjectsand failed most of them, losing his scholarship in the process. Ramanujan failed his Fellow of Arts exam in December 1906 and left the college without a degree but continued his independent research in mathematics. At this point in his life, he faced extreme poverty and starvation.

ATTENTION TOWARDS MATHEMATICS

On 14 July 1909, Ramanujan was married to a ten-year old bride, Janakiammal (21 March 1899 – 13 April 1994). After the marriage, Ramanujanfaced surgical operation and subsequent health problems. Recovering all these, he met deputy collector V. RamaswamyAiyer, who had recently founded the Indian Mathematical Society. Ramanujan, with the help of RamaswamyAiyer, had his work published in the *Journal of the Indian Mathematical Society*.

One of the first problems he posed in the journal was to find out the value of:

$$\sqrt{1+2\sqrt{1+3\sqrt{1+\cdots\ldots\ldots}}}$$

He waited for a solution to his problem in three issues, but failed to receive any. At the end, Ramanujan supplied the solution to the problem himself. He formulated an equation that could be used to solve the infinitely nested radicals' problem as

$$x+n+a=\sqrt{ax+(n+a)^2+x\sqrt{a(x+n)+(n+a)^2+(x+n)\sqrt{\ldots\ldots}}}$$

Using this equation, the answer to the question posed in the *Journal* was simply 3. Ramanujan wrote his first formal paper for the *Journal* on the properties of Bernoulli numbers. In his 17-page paper, "Some Properties of Bernoulli's Numbers", Ramanujan gave three proofs, two corollaries and three conjectures.

Ramanujan later wrote another paper and also continued to provide problems in the *Journal*. In early 1912, he got a temporary job in the Madras Accountant General's office, with a salary of 20 rupees per month which lasted for only a few weeks. Then, he applied for a position under the Chief Accountant of the Madras Port Trust. In a letter dated 9 February 1912, Ramanujan wrote: "Sir, I understand there is a clerkship vacant in your office, and I beg to apply for the same. I have passed the Matriculation Examination and studied up to the F.A. but was prevented from pursuing my studies further owing to several untoward circumstances. I have, however, been devoting all my time to Mathematics and developing the subject. I can say I am quite confident I can do justice to my work if I am appointed to the post. I therefore beg to request that you will be good enough to confer the appointment on me." Three weeks after he had applied, he was accepted as a Grade IV accounting clerk with a salary of 30 rupees per month. At his office, Ramanujan easily and quickly completed the assigned work and spent his spare time doing mathematical research.

CONTACTING ENGLISH MATHEMATICIANS

On 16 January 1913, Ramanujan wrote to G. H. Hardy. Coming from an unknown mathematician, the nine pages of mathematics made Hardy initially view Ramanujan's manuscripts as a possible "fraud". Hardy recognized some of Ramanujan's formulae but others seemed scarcely possible to believe. One of the theorems Hardy found so incredible was found on the bottom of page three (valid for $0 < a < b + 1/2$):

$$\int_0^\infty \frac{1+x^2/(b+1)^2}{1+x^2/a^2} \times \frac{1+x^2/(b+2)^2}{1+x^2/(a+1)^2} \times \dots\dots\dots\dots dx = \frac{\sqrt{\pi}}{2} \times \frac{\sqrt{(a+\frac{1}{2})}\sqrt{(b+1)}\sqrt{(b-a+\frac{1}{2})}}{\sqrt{(a)}\sqrt{(b+\frac{1}{2})}\sqrt{(b-a+1)}}$$

Hardy was also impressed by some of Ramanujan's other work relating to infinite series:

$$1-5\left(\frac{1}{2}\right)^3+9\left(\frac{1\times 3}{2\times 4}\right)^3-13\left(\frac{1\times 3\times 5}{2\times 4\times 6}\right)^3+\cdots\dots\dots\dots.. = \frac{2}{\pi}$$

$$1+9\left(\frac{1}{4}\right)^4+17\left(\frac{1\times 5}{4\times 8}\right)^4+25\left(\frac{1\times 5\times 9}{4\times 8\times 12}\right)^4+\cdots\dots\dots\dots.. = \frac{2^{\frac{3}{2}}}{\pi^{\frac{1}{2}2}\sqrt{\left(\frac{3}{4}\right)}}$$

The first result had already been determined by a mathematician named Bauer. The second one was new to Hardy, and was derived from a class of functions called a hypergeometric series which had first been researched by Leonhard Euler and Carl Friedrich Gauss. After he saw Ramanujan's theorems on continued fractions on the last page of the manuscripts, **Hardy commented that "they (theorems) defeated me completely; I had never seen anything in the least like them before"**. He figured that Ramanujan's theorems "must be true, because, if they were not true, no one would have the imagination to invent them". Hardy asked a colleague, J. E. Littlewood, to take a look at the papers. Littlewood was amazed by the mathematical genius of Ramanujan. After discussing the papers with Littlewood, Hardy commented that Ramanujan was "a mathematician of the highest quality, a man of altogether exceptional originality and power". One colleague, E. H. Neville, later commented that "not one (theorem) could have been set in the most advanced mathematical examination in the world".On 8 February 1913, Hardy wrote a letter to Ramanujan, expressing his interest for his work. After continued attempts both by Hardy and the renowned Mathematicians in India, Ramanujan agreed to visit England to work with Hardy and Littlewood.

LIFE IN ENGLAND

Ramanujan reached London on 14 April and immediately began his work with Littlewood and Hardy. Hardy and Ramanujan began to take a look at Ramanujan's notebooks. Hardy had already received 120 theorems from Ramanujan in the first two letters, but there were many more results and theorems to be found in the notebooks. Hardy saw that some were wrong, others had already been discovered, while the rest were new breakthroughs. Ramanujan left a deep impression on Hardy and Littlewood. Littlewood commented, "I can believe that he's at least a Jacobi", while Hardy said he "can compare him only with Euler or Jacobi."

Ramanujan spent nearly five years in Cambridge collaborating with Hardy and Littlewood and published a part of his findings there. While in England, Hardy tried his best to fill the gaps in Ramanujan's education without interrupting him.Ramanujan was awarded a B.A. degree by research (this degree was later renamed Ph.D.) in March 1916 for his work on highly composite numbers, the first part of which was published as a paper in the *Proceedings of the London Mathematical Society*. The paper was over 50 pages with different properties of such numbers proven. Hardy remarked that this was one of the most unusual papers seen in mathematical research at that time and that Ramanujan showed extraordinary ingenuity in handling it. On 6 December 1917, he was elected to the London Mathematical Society. He became a Fellow of the Royal Society in 1918 having the proud of being one of the youngest Fellows in the history of the Royal Society. On 13 October 1918, he was elected a Fellow of Trinity College, Cambridge for his investigation in Elliptic functions and the Theory of Numbers.

CONTRIBUTION OF RAMANUJAN IN MATHEMATICS

In mathematics, there is a distinction between having an insight and having a proof. Ramanujan's talent suggested a plethora of formulae that provided a strong base for further mathematical research. It is said that Ramanujan's discoveries are unusually rich and that there is often more to them than initially seen. As a by-product, new directions of research were opened up. Examples of the most interesting of these formulae include the intriguing infinite series for π, one of which is given below

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{k=0}^{\infty} \frac{(4k)!\,(1103 + 26390k)}{(k!)^4 396^{4k}}.$$

This result is based on the negative fundamental discriminant $d = -4\times58 = -232$ with class number $h(d) = 2$ (note that 5×7×13×58 = 26390 and that 9801=99×99; 396=4×99) and is related to the fact that

$$e^{\pi\sqrt{58}} = 396^4 - 104.000000177 \ldots \ldots \ldots$$

Ramanujan's series for π converges extraordinarily rapidly (exponentially) and forms the basis of some of the fastest algorithms currently used to calculate π. Truncating the sum to the first term also gives the approximation$\frac{9801\sqrt{2}}{4412}$ for π, which is correct to six decimal places.

One of his remarkable capabilities was the rapid solution for problems. He was sharing a room with P. C. Mahalanobis who had a problem, "Imagine that you are on a street with houses marked 1 through n. There is a house in between (x) such that the sum of the house numbers to left of it equals the sum of the house numbers to its right. If n is between 50 and 500, what are n and x?" This is a bivariate problem with multiple solutions. Ramanujan thought about it and gave the answer with a twist: He gave a continued fraction. The unusual part was that it was the solution to the whole class of problems. Mahalanobis was astounded and asked how he did it. "It is simple. The minute I heard the problem, I knew that the answer was a continued fraction. Which continued fraction, I asked myself. Then the answer came to my mind," Ramanujan replied. His intuition also led him to derive some previously unknown identities, such as

$$\left[1 + 2\sum_{n=1}^{\infty} \frac{\cos(n\theta)}{\cosh(n\pi)}\right]^{-2} + \left[1 + 2\sum_{n=1}^{\infty} \frac{\cosh(n\theta)}{\cosh(n\pi)}\right]^{-2} = \frac{2^4 \left(\frac{3}{4}\right)}{\pi}$$

for all θ, where (z) is the gamma function. Expanding into series of powers and equating coefficients of θ^0,θ^4 and θ^8 gives some deep identities for the hyperbolic secant.

In 1918, Hardy and Ramanujan studied the partition function $P(n)$ extensively and gave a non-convergent asymptotic series that permits exact computation of the number of partitions of an integer. Hans Rademacher, in

1937, was able to refine their formula to find an exact convergent series solution to this problem. Ramanujan and Hardy's work in this area gave rise to a powerful new method for finding asymptotic formulae, called the circle method. He discovered mock theta functions in the last year of his life. For many years these functions were a mystery, but they are now known to be the holomorphic parts of harmonic weak Maass forms.

THE RAMANUJAN CONJECTURE

Although there are numerous statements that could have borne the name *Ramanujan conjecture*, there is one statement that was very influential on later work. In particular, the connection of this conjecture with conjectures of André Weil in algebraic geometry opened up new areas of research. That Ramanujan conjecture is an assertion on the size of the Tau-function, which has as generating function the discriminant modular form $\Delta(q)$, a typical cusp form in the theory of modular forms. It was finally proven in 1973, as a consequence of Pierre Deligne's proof of the Weil conjectures. The reduction step involved is complicated. Deligne won a Fields Medal in 1978 for his work on Weil conjectures.

RAMANUJAN'S NOTEBOOKS

Ramanujan recorded the bulk of his results in four notebooks of loose leaf paper. These results were mostly written up without any derivations. This is probably the origin of the misperception that Ramanujan was unable to prove his results and simply thought up the final result directly. Mathematician Bruce C. Berndt, in his review of these notebooks and Ramanujan's work, says that Ramanujan most certainly was able to make the proofs of most of his results, but chose not to do so.This style of working may have been for several reasons. Since paper was very expensive, Ramanujan would do most of his work and perhaps his proofs on slate, and then transfer only the results to paper. He was also quite likely to have been influenced by the style of G. S. Carr's book studied in his youth, which stated results without proofs. Finally, it is possible that Ramanujan considered his workings to be for his personal interest alone; and therefore recorded only the results. Professor Bruce C. Berndt of the University of Illinois, during a lecture at IIT Madras in May 2011, stated that over the last 40 years, almost all of Ramanujan's theorems have been proven right.

The first notebook has 351 pages with 16 somewhat organized chapters and some unorganized material. The second notebook has 256 pages in 21 chapters and 100 unorganised pages, with the third notebook containing 33 unorganised pages. The results in his notebooks inspired numerous papers by later mathematicians trying to prove what he had found. Hardy himself created papers exploring material from Ramanujan's work as did G. N. Watson, B. M. Wilson, and Bruce Berndt. A fourth notebook with 87 unorganised pages, the so-called "lost notebook", was rediscovered in 1976 by George Andrews. Notebooks 1, 2 and 3 were published as a two-volume set in 1957 by the Tata Institute of Fundamental Research (TIFR), Mumbai, India. This was a photocopy edition of the original manuscripts, in his own handwriting.

RAMANUJAN-HARDY NUMBER 1729

The number 1729 is known as the Hardy–Ramanujan number after a famous anecdote of the British mathematician G. H. Hardy regarding a visit to the hospital to see Ramanujan. In Hardy's words: I remember once going to see him when he was ill at Putney. I had ridden in taxi cab number 1729 and remarked that the number seemed to me rather a dull one. "No," he replied, "it is a very interesting number; it is the smallest number expressible as the sum of two cubes in two different ways." The two different ways are

$1729 = 1^3+12^3 = 9^3+10^3$. This shows that how Ramanujan used to love with the numbers and to what extent he was a magician of number theory.

CONCLUSIONS

Throughout the discussion, it is very clear that the genius like SrinivasaRamanujan is rarely born. He was a person with inbuilt talent with no comparisons with anyone else. In a very short span of 32 years of life, his contribution in the development of Mathematics not only surprised the Mathematicians of that time but also provided a path to future researchers. The Great Ramanujan will always be remembered for his extraordinary contribution in the field of mathematical analysis, number theory, infinite series and continued fractions.

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