**STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086**
**(For candidates admitted during the academic year 2009 – 10)**

**SUBJECT CODE : MT/PE/SP33**

**M. Sc. DEGREE EXAMINATION, NOVEMBER 2010**
**BRANCH I - MATHEMATICS**
**THIRD SEMESTER**

**COURSE : ELECTIVE**
**PAPER : STOCHASTIC PROCESSES**
**TIME : 3 HOURS** **MAX. MARKS : 100**

**SECTION – A** **( 5 X 8 = 40 )**

**ANSWER ANY FIVE QUESTIONS**

1. For a Poisson process N(t), show that
   $P_n\ (t) = \frac{e^{-\lambda t}(\lambda t)^n}{n!}$ where $P_n(t) = P\{N(t) = n\},\ for\ \ n = 0,1,2,\dots$ .
2. Find the density function of $S_n$ , the arrival time of $n^{th}$ event. Assume that the process probabilistically restart itself.
3. Define Compound Poisson random variable. Find its moment generating function, mean and variance.
4. Define Renewal function m(t). Show that $m(t) = \sum_{n=1}^{\infty} F_n(t)$.
5. Show that with probability 1, $\frac{N(t)}{t} \to \frac{1}{\mu}$ as $t \to \infty$.
6. Find the limiting probabilities for the embedded M/G/1 queue.
7. Explain simple epidemic model. If T denote the time until the total population is infected, find E[T] and Var[T].

**SECTION – B** **( 3 X 20 = 60 )**

**ANSWER ANY THREE QUESTIONS**

8. If $N_i(t)$ represents the number of type-I events that occur by time t, i=1,2, then $N_1(t)$ and $N_2(t)$ are independent Poisson random variables having respective means λp and λ(1-p), where
   $$p = \frac{1}{t}\int_0^t P(s)ds.$$
9. If $E[R] < \infty$ and $E[X] < \infty$, Prove that
   (i) With probability 1, $\frac{R(t)}{t} \to \frac{E[R]}{E[X]}$ as $t \to \infty$

   (ii) $\frac{E[R(t)]}{t} \to \frac{E[R]}{E[X]}$ as $t \to \infty$

10. For a Poisson process {N(t), t ≥0},

show that $P\{N(s) = k \;\; / \;\; N(t) = n\} = \binom{n}{k}\left(\frac{s}{t}\right)^k\left(1-\frac{s}{t}\right)^{n-k}$ for $k = 0,1, \dots, n.$

Also, calculate $E[N(t).N(t+s)]$.

11. For Yule process, show that $P_{ij}(t) = \binom{j-1}{i-1}e^{-\lambda i t}(1-e^{-\lambda t})^{j-1}, j \geq i \geq 1.$

12. For all i,j and t$\geq$ 0, prove the Kolmogorov backward equations,

$P_{ij}^{'}(t) = \sum_{k\neq i} q_{ik} \quad P_{kj}(t) - v_i P_{ij}(t).$

Also, write the corresponding equations for two-state chain.

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