STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2009 – 10 & thereafter)

SUBJECT CODE : MT/PC/RA14

M. Sc. DEGREE EXAMINATION, NOVEMBER 2010
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : CORE
PAPER : REAL ANALYSIS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 X 8 = 40)**

ANSWER ANY FIVE QUESTIONS

1. State and prove "The cantor intersection theorem".

2. a) Let (S, d)be a metric subspace of (M, d) and let X be a subset of S, then prove that X is open in S if and only if $X = A \cap S$ for some set A open in M.
   b) Prove that X is compact where X is a closed subset of a compact metric space M.

3. a) Justify the following statement with a suitable example "the operations of limit and integration cannot always be interchanged".
   b) State the Cauchy condition for uniform convergence of series.

4. a) Define orthogonal and orthonormal system of functions.
   b) Verify that the trigonometric system $S = \{\varphi_0, \varphi_1 \dots\}$ where $\varphi_0(x) = \frac{1}{\sqrt{2\pi}}$, $\varphi_{2n-1}(x) = \frac{\cos nx}{\sqrt{\pi}}$, $\varphi_{2n}(x) = \frac{\sin nx}{\sqrt{\pi}}$ is orthonormal on $[0, 2\pi]$.

5. State and prove Riemann Lebesgue lemma.

6. a) Let $f: \mathbb{R}^2 \to \mathbb{R}^3$ be defined by the equation $f(x, y) = (\sin x \cos y\,,\ \sin x \sin y\,,\ \cos x \cos y)$. Determine the Jacobian matrix $\bar{D}\bar{f}(x, y)$.
   b) State and prove the Taylor's formula for function from $\mathbb{R}^n$ to $\mathbb{R}^1$.

7. a) Define a saddle point.
   b) Find the maxima and minima of the function $x^3 + y^3 - 3x - 12y + 20 = 0$

SECTION – B **(3 X 20 = 60)**

ANSWER ANY THREE QUESTIONS

8. a) Prove that (i) the union of any collection of open sets is an open set.
   (ii) a set S in $\mathbb{R}^n$ is closed if and only if it contain all its adherent points

   b) Let S be a subset of $\mathbb{R}^n$ then prove that the following three statements are equivalent.
   (i) S is compact
   (ii) S is closed and bounded
   (iii) Every infinite subset of S has an accumulation point in S

9. a) Distinguish between pointwise convergence and uniform convergence.
   b) State and prove Cauchy condition for uniform convergence.

10. a) State and prove Jordan's theorem.
    b) Obtain an integral representation for the partial sum of a fourier series.

11. a) State and prove the chain rule for derivatives.
    b) Derive a sufficiency condition for equality of mixed partial derivatives.

12. State and prove the Inverse function theorem.