STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2009 – 10)

SUBJECT CODE: MT/PC/PD34

M. Sc. DEGREE EXAMINATION, NOVEMBER 2010
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : CORE
PAPER : PARTIAL DIFFERENTIAL EQUATIONS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 X 8 = 40)**

ANSWER ANY FIVE QUESTIONS

1. (i) Eliminate the arbitrary function f from the relation $Z = xy + f(x^2 + y^2)$

   (ii) Find the general solution of the linear partial differential equation

$$z(xp - yq) = y^2 - x^2$$

2. Solve the equation

$$\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} - \frac{\partial^3 z}{\partial x \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}$$

3. Classify and reduce the following equations to canonical form and solve it

$$3u_{xx} + 10u_{xy} + 3u_{yy} = 0$$

4. Solve the two-dimensional Laplace equation

   $\nabla^2 u = u_{xx} + u_{yy} = 0$ by using variable seperable method.

5. The ends A and B of a rod, 10cm in length are kept at temperature 0°C and 100°C respectively until the steady state condition prevails. Suddenly the temperature at the end *A* is increased to 20°C and the end B is decreased to 60°C. Find the temperature distribution in rod at time *'t'*.

6. State and prove the uniqueness theorem for the heat conduction equation.

7. Derive the one-dimensional wave equation.

SECTION – B **(3 X 20 = 60)**

ANSWER ANY THREE QUESTIONS

8. (a) Find the complete integral of

   $x^2p^2 + y^2q^2 - 4 = 0$ using charpit's method.

   (b) Show that the following PDEs

   $xp - yq = x$ and $x^2p + q = xz$ are compatible and hence, find their solution.

9. (a) Find the solution of the equation $\nabla_1^2 z = e^{-x} \cos y$ which tends to zero as $x \to \infty$ and has the value $\cos y$ when $x = 0$

(b) Reduce the PDE

$$y^2 \frac{\partial^2 z}{\partial x^2} - 2xy \frac{\partial^3 z}{\partial x \ \partial y} + x^2 \frac{\partial^3 z}{\partial y^2} = \frac{y^2}{x} \frac{\partial z}{\partial x} + \frac{x^2}{y} \frac{\partial z}{\partial y}$$

to canonical form and solve it.

10. (a) Derive the poisson equation

(b) Find the solution of Interior Dirichlet problem for a circle.

11. (a) Show that the solution of the equation $\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}$

satisfying the conditions (i) $T \to 0$, as $t \to \infty$

(ii) $T = 0$, for $x = 0$ and $x = a$ for all $t > 0$

(iii) $T = x$, when $t = 0$ and $0 < x < a$

is $T(x, t) = \frac{2a}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \sin\left(\frac{n\pi}{a} x\right) \exp\left[-\left(\frac{n\pi}{a}\right)^2 t\right]$

(b) Solve the Diffusion equation in cylindrical coordinates.

12. (a) State and prove D'Alemberts solution of one-dimensional wave equation.

(b) Obtain the solution of the Radio equation $\frac{\partial^2 v}{\partial x^2} = LC \frac{\partial^2 v}{\partial t^2}$

Appropriate to the case when a periodic e.m.f. $v_o \cos pt$ is applied at the end $x = 0$ of the line.

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