STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086 (For candidates admitted during the academic year 2009 – 10)

SUBJECT CODE: MT/PC/MS34

M. Sc. DEGREE EXAMINATION, NOVEMBER 2010 BRANCH I - MATHEMATICS THIRD SEMESTER

COURSE : CORE

PAPER : MATHEMATICAL STATISTICS

TIME : 3 HOURS MAX. MARKS : 100

SECTION – A $(5 \times 8 = 40)$ ANSWER ANY FIVE QUESTIONS

- 1. Find the characteristic function of a Poisson random variable *X* and hence obtain its mean and variance.
- 2. Define Beta distribution and obtain its moments.
- 3. State and prove Chebyshev's Theorem.
- 4. Derive Chi-Square distribution. Find its mean and variance.
- 5. Define sample mean. Obtain the distribution of the Arithmetic Mean of independent normally distributed random variables.
- 6. Prove that \bar{X} is an unbiased estimate of mean m and s^2 is not unbiased estimate of variance σ^2 .
- 7. Explain the method of maximum likelihood.

SECTION – B $(3 \times 20 = 60)$ ANSWER ANY THREE QUESTIONS

- 8. State and prove Levy's inversion theorem.
- 9. (i) Prove that a random variable will have one point distribution if and only if its variance is zero.
 - (ii) Obtain the characteristic function of random variable with Gamma distribution. Check the validity of addition theorem for gamma random variable.
- 10. (i) State and prove Lindeberg Levy theorem.
 - (ii) State and prove Bernoulli's law of large numbers.
- 11. Derive students t-distribution. What is the condition for kth order moment to exits?
- 12. Prove that an unbiased estimate U of the parameter Q is the most efficient if and only if (i) the estimate U is sufficient.
 - (ii) for g(U,Q) > 0, the density g(U,Q) almost everywhere satisfies the relation

$$\frac{\partial \log g(U,Q)}{\partial Q} = c(U-Q).$$