STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2009 – 10 & thereafter)

SUBJECT CODE : MT/PC/MA14

M. Sc. DEGREE EXAMINATION, NOVEMBER 2010
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : CORE
PAPER : MODERN ALGEBRA
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 X 8 = 40)**
ANSWER ANY FIVE QUESTIONS

1. Prove that the number of p-Sylow's subgroups of a finite group is of the form 1+kp.
2. State and prove the Eisenstein irreducibility criterion of a polynomial with integer coefficients.
3. Define a module over a ring. Prove that any abelian group can be considered as an module over the ring of integers.
4. Prove that a polynomial of degree 'n' over a field can have at most 'n' roots in any extension field.
5. Prove that for every prime p and for every positive integer m there exists a field with p^m elements.
6. If F is a finite field and $\alpha \neq 0, \beta \neq 0$ are two elements of F then prove that there exists elements a and b in F such that $1+\alpha a^2+\beta b^2=0$.
7. Define a perfect field and prove that a field of characteristic zero is perfect.

SECTION – B **(3 X 20 = 60)**
ANSWER ANY THREE QUESTIONS

8. If 'p' is a prime number and if $p^\alpha \mid o(G)$, then prove that G has a subgroup of order p^α.
9. Prove that the polynomial ring F[X] is a Euclidean ring.
10. Prove that any finitely generated module over a Euclidean ring is a direct sum of a finite number of cyclic submodules.
11. Prove that an element $a \in K$ is algebraic over F if and only if F(a) is a finite extension of F.
12. Prove that a finite division ring is a field.