STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2009 – 10)

SUBJECT CODE : MT/PC/CM34

M. Sc. DEGREE EXAMINATION, NOVEMBER 2010
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : CORE
PAPER : CONTINUUM MECHANICS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 X 8 = 40)**

ANSWER ANY FIVE QUESTIONS

1. Establish the relation between stress tensor and the stress vector.

2. Determine the principal stress values and principal direction for the stress tensor $\sigma_{ij} = \begin{pmatrix} \tau & \tau & \tau \\ \tau & \tau & \tau \\ \tau & \tau & \tau \end{pmatrix}$.

3. The Langrangian description of a deformation is given by $x_1 = X_1 + X_3\left(e^2 - 1\right), x_2 = X_2 + X_3\left(e^2 - e^{-2}\right), x_3 = e^2 X_3$ where e is a constant .show that the Jacobian J does not vanish and determine the Eulerian equations describing this motion.

4. A linear (small strain) deformation is specified by $u_1 = 4x_1 - x_2 + 3x_3, u_2 = x_1 + 7x_2, u_3 = -3x_1 + 4x_2 + 4x_3$. determine the principal strains and the principal deviator strains for these deformation.

5. For the steady velocity field $v = 3x_1^2 x_2 \hat{e}_1 + 2x_2^2 x_3 \hat{e}_2 + x_1 x_2 x_3^2 \hat{e}_3$, determine the rate of extension at $P(1,1,1)$ in the direction of $\hat{r} = (3\hat{e}_1 - 4\hat{e}_3)/5$.

6. A two- dimensional incompressible flow is given by $v_1 = A(x_1^2 - x_2^2)/r^4, v_2 = A(2x_1x_2)/r^4, v_3 = 0, where\ r^2 = x_1^2 + x_2^2$.show that the continuity equation is satisfied by this motion. Show also that the flow is irrational.

7. Determine the elastic coefficient matrix for a continuum having an axis of elastic symmetry of order N=4.Assume $C_{KM} = C_{MK}$.

SECTION – B **(3 X 20 = 60)**

ANSWER ANY THREE QUESTIONS

8. (a) Show that the Cauchy stress quadric for a state of stress represented by $\Sigma = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ is an ellipsoid (the stress ellipsoid) when a,b and c are all of the same sign.

   (b) The stress tensor at a point P is given with respect to the axes $Ox_1x_2x_3$ by the values $\sigma_{ij} = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$. Determine the principal stress values and the principal stress directions represented by the axes $Ox_1{}^*x_2{}^*x_3{}^*$

9. (a) A displacement field is given by $x_1 = X_1 + AX_2, x_2 = X_2 + AX_3, x_3 = X_3 + AX_1$ where A is a constant. Calculate the Lagrangian linear strain tensor L and the Eulerian linear strain tensor E. Compare L and E for the case when A is very small.

   (b) Under the restriction of a small deformation theory, $L = E$. Accordingly for a displacement field given by $u = (x_1 - x_3)^2 \hat{e}_1 + (x_2 + x_3)^2 \hat{e}_2 - x_1x_2 \hat{e}_3$, determine the linear strain tensor, the linear rotation tensor and the rotation vector at the point $P(0,2,-1)$.

10. (a) A velocity field is specified by the vector $v = x_1^2 t \hat{e}_1 + x_2 t^2 \hat{e}_2 + x_1 x_3 t \hat{e}_3$. Determine the velocity and acceleration of the particle at $P(1,3,2)$ when $t = 1$.

    (b) Define a vortex line. Show that the equations for vortex lines are $\frac{dx_1}{q_1} = \frac{dx_2}{q_2} = \frac{dx_3}{q_3}$

    (c) For the steady velocity field $v_1 = x_1^2 x_2 + x_2^3,\ v_2 = -x_1^3 - x_1 x_2^2, v_3 = 0$ determine expressions for the principal values of the rate of deformation tensor D at an arbitrary point $P(x_1, x_2, x_3)$.

11. (a) Using linear momentum principle, obtain the equation of motion of a moving continuum.

    (b) For the velocity field $v_i = x_i/(1+t)$, Show that $\rho^{x_1 x_2 x_3} = \rho^{X_1 X_2 X_3}$.

12. Obtain the generalized Hooke's law for an isotropic body in terms of the elastic constants λ and μ. Show that the strain energy density function u* for an isotropic Hookean solid may be expressed in terms of the strain tensor by $u^* = \frac{\lambda}{2}\varepsilon_{ii}\varepsilon_{jj} + \mu\varepsilon_{ij}\varepsilon_{ij}$.

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