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CREDIT MARKET DISTORTIONS, ASSET PRICES AND MONETARY POLICY

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We study the conditions that ensure rational expectations equilibrium (REE) determinacy and expectational stability (E-stability) in a standard sticky-price model augmented with the cost channel. We allow for varying degrees of pass-through of the policy rate to bank-lending rates. Strong cost-side effects limit the size of the policy rate response to inflation that is consistent with determinacy, so that inflation-targeting policies may not be capable of ensuring REE uniqueness. In this case it is advisable to combine policy rate responses to inflation with an appropriate reaction to the output gap and/or firm profitability. The negative reaction of real activity and asset prices to inflationary shocks adds a negative force to inflation responses that counteracts the borrowing cost effect and prevents expectations of higher inflation from becoming self-fulfilling.

Keywords: Monetary Policy, Cost Channel, Asset Prices, Determinacy, E-stability

1. INTRODUCTION

Financial intermediation and corporate credit play a central role in the transmission of monetary policy, determining the impact of interest rate changes on the prices of goods and assets. This paper examines a dynamic general equilibrium model in which bank-lending shapes the transmission of monetary policy to firm profitability, through the so-called cost channel.[1] We characterize the conditions that ensure

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rational expectations equilibrium (REE) uniqueness and stability under adaptive learning (E-stability) in the presence of varying degrees of pass-through from policy to bank-lending rates. Furthermore, we explore the interplay between asset prices and cost-side effects, as well as the implications of allowing the monetary authority to respond to asset prices, to enhance both price and financial stability.

Monetary policy supply-side effects classically arise from agency problems between producers and lenders. The importance of this channel crucially depends on the pass-through from policy to bank-lending rates. Chowdhury et al. (2006) show that heterogeneous financial systems can lead to major differences in the transmission of policy shocks: along with countries where the banking sector acts as an attenuator of changes in the risk-free rate (e.g., France and Germany), there are countries where bank-lending rates amplify movements in the policy rate (e.g., Japan and the United States). The second case is central to our analysis.

Along with the traditional Taylor principle, the cost channel implies the emergence of an upper bound to inflation responses that prevents the central bank from being too aggressive in stabilizing inflation, if determinacy is to be attained. In contrast to previous studies [e.g., Brückner and Schabert (2003), Surico (2008), and Llosa and Tuesta (2009)], we show that the additional constraint may become a reason of concern for the policy maker when movements in the policy rate are amplified by the banking sector. In fact, the upper frontier may become so stringent that determinacy cannot be attained if the central bank acts as a pure inflation targeter. Setting the policy rate in response to both inflation and the output gap may be desirable in these circumstances. Reacting to real activity reduces the overall interest rate response to inflation, thus counteracting the borrowing cost effect that operates through the direct influence of the lending rate on aggregate supply.

A main focus of this paper is on examining the role of monetary policy when the cost channel "matters" and affects firm profitability. Despite the increasing emphasis on the connection between financial frictions and macroeconomic fluctuations, the influence of cost-side effects on firm profits and asset prices has generally been neglected. We show that responding to asset prices helps in attaining determinacy when strong credit market distortions are at work. With regard to this, two distinct effects are isolated. On one hand, as shown by Carlstrom and Fuerst (2007), reacting to asset prices weakens the overall policy response to inflationary shocks, thus making the lower bound to inflation responses more stringent. On the other hand, a positive response to asset prices brings about much higher gains, increasing the probability of attaining determinacy and outweighing the borrowing cost effect that operates in the model with the cost channel. In turn, the second effect emerges as the outcome of two mutually reinforcing mechanisms that exploit the amplification induced by strong degrees of pass-through from policy to bank lending rates. To see this, consider an increase in the nominal rate of interest aimed at offsetting the inflationary consequences of a demand or supply shock. When the central bank adjusts the policy rate in response to asset price misalignments, the negative deviation of firm profits from their level under flexible prices exerts a direct impact on inflation that counteracts the borrowing cost effect on aggregate supply. In addition, the cost channel implies a direct influence of interest rate changes on firm dividends that dominates the negative correlation between the output gap and the dividend gap that generally arises in models with nominal rigidities. Together, these mechanisms prevent expectations of higher inflation from becoming self-fulfilling.

The remainder of the paper is laid out as follows: Section 2 introduces the theoretical setting; Section 3 shows that implementing rules that are exclusively aimed at stabilizing inflation may never ensure determinacy and E-stability in the presence of strong cost-side effects; Section 4 explores the connection between firm profitability and the cost channel, and shows that adjusting the policy rate in response to asset prices misalignments may help in alleviating the problems of dynamic instability highlighted in the previous section. The last section concludes.

2. THE MODEL

The model economy is populated by households, firms, and financial intermediaries. Households have preferences defined over a variety of consumption goods, supply labor to monopolistically competitive firms, and deposit funds at the financial intermediaries. Firms utilize labor to produce goods and borrow from financial intermediaries to finance the wage bill, which has to be paid out before revenues are collected. The decision problems of households and firms follow the standard treatment of Ravenna and Walsh (2006) and are outlined in Pfajfar and Santoro (2012). This section describes the role of financial intermediaries and reports the log-linearized model economy.

2.1. Financial Intermediation

We assume that financial intermediaries receive deposits M_t^d (remunerated at the gross rate R_t) from households and a cash injection X_t from the monetary authority. Moreover, they supply loans L_t to firms at the (gross) nominal rate R_t^l. These funds are used to finance the wage bill, $W_t N_t$, where W_t and N_t denote the real wage and the labor input, respectively. Following Chowdhury et al. (2006), we allow varying degrees of interest rate changes to affect firms' cost of borrowing. For simplicity, we assume that this friction can be measured by an increasing function of the nominal rate of interest, $\Psi_t(R_t) \in (0, 1)$, which can be interpreted as a measure of defaults on loans.[2] Moreover, we assume an explicit cost to manage loans, which amounts to $\kappa^l (\geq 0)$ per unit of loan. Intermediaries operate in a competitive environment. Nominal profits in the banking sector are defined as

$$\Pi^{\text{int}} = R_t^l \left[1 - \Psi(R_t)\right] L_t - R_t M_t^d - \kappa^l L_t. \qquad \textbf{(1)}$$

The following bank balance sheet condition must hold in every period: $L_t = M_t^d + X_t$. From the maximization of (1) subject to this constraint, we obtain a relationship that links deposit and loan rates. In log-linear terms, this no-arbitrage condition reads as[3] $\widehat{r}_t^l = (1+\psi)\widehat{r}_t$, where $\psi (= \psi_1 - \psi_2)$ captures the elasticity of the contractual interest rate to percentage changes in the policy rate. This results from the combination of two components, $\psi_1 = [R\Psi'(R)/1 - \Psi(R)]$ and $\psi_2 = [\kappa^l/(R+\kappa^l)]$. A negative ψ indicates that a change in the risk-free interest rate is not completely passed through to the lending rate. This is the case when managing costs are too high, and the cost channel is mitigated. Alternatively, Hannan and Berger (1991) attribute this effect to loan price rigidities.[4] When ψ is positive, a rise in the policy rate is even accelerated, so that the lending rate rises by more than one to one. This can be viewed as a reduced-form relation based on financial market imperfections stemming from asymmetric information between borrowers and lenders, as advocated by the literature on the financial accelerator [see Bernanke et al. 1999)]. Note that both $\psi < 0$ and $\psi > 0$ are empirically relevant cases: Chowdhury et al. (2006) estimate $\psi = -0.8$ for France and $\psi = -0.04$ for Germany, whereas they report $\psi = 0.5$ for Italy, $\psi = 0.1$ for Canada, and $\psi = 0.3$ for the United States and United Kingdom.

2.2. Log-Linear System

The first-order conditions characterizing the decisions of households and firms are log-linearized around the steady state. The linearized economy features an *IS* curve and an aggregate short-run aggregate supply (AS) schedule,

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}(r_t - E_t \pi_{t+1}), \quad \textbf{(2)}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa (1+\psi) r_t + \kappa (\sigma + \eta) y_t + \varepsilon_t, \quad \textbf{(3)}$$

where y_t is the output gap, π_t is the rate of inflation, r_t is the nominal interest rate gap, and ε_t is a cost-push term that derives from assuming a log-stationary process from the elasticity of substitution in consumption, θ.[5] Moreover, β denotes the households' discount factor, σ is the inverse of the elasticity of intertemporal substitution, η is the inverse of the Frisch elasticity of labor supply, and $\kappa = (1-\omega\beta)(1-\omega)\omega^{-1}$, where $1-\omega$ is the probability that firms can reoptimize their prices at each given period, as in Calvo (1983).

From the households' optimization problem we also retrieve a linearized relationship that describes the evolution of the asset price gap, q_t:

$$q_t = (1-\beta)\, d_t + \beta E_t q_{t+1} - \beta (r_t - E_t \pi_{t+1}), \quad \textbf{(4)}$$

where d_t is the dividend gap. We assume that profits are fully transferred to the stockholders, so that dividends are $D_t = Y_t - R_t^l W_t N_t$ and the log-linear dividend gap reads as $d_t = \varsigma y_t - \mu r_t$, where $\varsigma = 1-(\theta-1)(\sigma+\eta)$ and $\mu = (\theta-1)(1+\psi)$. Note that ς is negative for a wide range of plausible parameterizations. The negative relationship between d_t and y_t is a characteristic feature of models with nominal rigidities and it represents the key to explain why adjusting the policy rate in response to movements in the price of assets may harm dynamic stability when only the demand channel of the monetary transmission mechanism is at work.[6] A novel feature in the specification of the dividend gap lies in the direct influence of the interest rate gap on d_t. The magnitude of this effect—which is captured by μ—increases in the degree of pass-through and is paramount to explain why responding to firm profitability may increase the probability to attain determinacy in the presence of strong supply-side effects. Section 4 will explore this point in close detail. We plug the dividend gap into (4) to get

$$q_t = \beta E_t q_{t+1} + \beta E_t \pi_{t+1} + \epsilon y_t - \xi r_t, \tag{5}$$

where $\epsilon = (1-\beta)\varsigma$ and $\xi = \beta + (1-\beta)\mu$. Coefficient ξ suggests that, compared to the baseline setting with no cost channel, the effect of r_t on the asset price gap is reinforced.[7]

3. DETERMINACY AND E-STABILITY UNDER BENCHMARK INTEREST RATE RULES

This section explores rational expectations equilibrium determinacy and stability under adaptive learning (E-stability). To close the model, we alternatively consider Taylor-type rules that differ with respect to their timing and the information set available to the policy maker. The central bank may adjust the nominal rate of interest in response to movements in current (or expected) inflation, the output gap, and asset prices misalignments from their equilibrium under flexible prices. To build some intuition on the interplay between monetary policy and firm profitability in the presence of cost-side effects, we find it instructive to first explore rules whereby the policy maker pursues nominal and real stability (Section 3), while postponing to Section 4 the analysis of rules that account for the dynamics of asset prices.

After a specific policy rule is substituted into (2), (3), and (5), the resulting model can be reported as

$$\mathbf{\Gamma x}_t = \mathbf{\Phi} + \mathbf{\Omega} E_t \mathbf{x}_{t+1} + \mathbf{\Xi} \boldsymbol{\varpi}_t, \tag{6}$$

$$\boldsymbol{\varpi}_t = \boldsymbol{\rho} \boldsymbol{\varpi}_{t-1} + \boldsymbol{\epsilon}_t, \tag{7}$$

where $\mathbf{x}_t = [\pi_t, y_t, q_t]'$, $\boldsymbol{\varpi}_t$ is a vector of shocks, and $\mathbf{\Gamma}$, $\mathbf{\Omega}$, and $\mathbf{\Phi}$ are matrices of structural parameters. Exogenous variables are assumed to follow a first-order stationary VAR with i.i.d. innovations and diagonal covariance matrix. It can be shown that REE uniqueness obtains if and only if the eigenvalues of $\mathbf{\Gamma}^{-1}\mathbf{\Omega}$ have real parts lying in the unit circle. Moreover, a necessary and sufficient condition for E-stability is that $\mathbf{J}(= \mathbf{\Gamma}^{-1}\mathbf{\Omega} - \mathbf{I})$ has all roots with negative real parts [see Evans and Honkapohja (2001)].

The model (6), (7) falls within the class considered by McCallum (2007), according to which determinacy is a sufficient (though not necessary) condition for E-stability. Therefore, unique evolutionarily stable RE solutions detected in this framework retain the property of E-stability. Moreover, we will also observe cases characterized by both indeterminacy and E-stability. These nonfundamental solutions will be briefly discussed, although most of the analysis will be restricted to the study of the fundamental solutions (i.e., MSV-type solutions). In addition, Section 3.2 will consider a rule based on expectations of contemporaneous data. It should be noted that under "nowcasting" the model features expectations of the endogenous state variables at both time t and $t+1$, so that (6) is replaced by $\mathbf{\Gamma}^n \mathbf{x}_t = \mathbf{\Phi} + \mathbf{\Lambda}^n E_t \mathbf{x_t} + \mathbf{\Omega}^n E_t \mathbf{x}_{t+1} + \Xi \varpi_t$, where "$n$" stands for nowcasting. Thus, the model does not belong to the class examined by McCallum (2007) and E-instability may even characterize determinate equilibria.

3.1. Contemporaneous Data Rules

We start the analysis by assuming a central bank that adjusts the rate of interest in response to movements in the contemporaneous rate of inflation (i.e., $r_t = \chi_\pi \pi_t$, $\chi_\pi > 0$). The following proposition formalizes the conditions for determinacy in connection with the magnitude of cost-side effects. As such, it retains considerable importance for monetary authorities that are primarily or exclusively concerned with inflation stability.[8]

PROPOSITION 1. *Under the contemporaneous data rule $r_t = \chi_\pi \pi_t$ the following conditions are necessary and sufficient to ensure REE determinacy: (i) $\chi_\pi > \overline{\chi}_\pi = 1$; (ii) iff $\psi > \frac{\eta}{\sigma}$: $\chi_\pi < \widehat{\chi}_\pi = \frac{\sigma(1-\beta)}{\kappa(\sigma\psi-\eta)}$; (iii) iff $\psi > \frac{\eta-\sigma}{2\sigma}$: $\chi_\pi < \widetilde{\chi}_\pi = \frac{2\sigma(1+\beta)+\kappa(\sigma+\eta)}{\kappa(\sigma(1+2\psi)-\eta)}$.*

Proof. See the Appendix. ■

To enhance a visual understanding of Proposition 1, we plot the conditions that ensure E-stability and determinacy. Following McCallum and Nelson (1999), we set $\sigma = 1/0.164$ and $\kappa = 0.3/(\sigma + \eta)$.[9] As to the other structural coefficients, we set $\beta = 0.99$, $\eta = 2$, and $\theta = 6$.

Figure 1 shows that a strong degree of pass-through may imply that determinacy is never attained if the central bank acts as a pure inflation targeter. This is due to the intersection of the upper and lower frontiers that determinacy imposes on inflation responses. Given the parameterization that we use, Proposition 1 allows us to compute numerical ranges of the pass-through coefficient that are characterized by different properties in terms of dynamic stability. In fact, equilibrium uniqueness can never be attained for $\psi > \frac{\eta}{\sigma} + \frac{(1-\beta)}{\kappa}$ (≈ 0.60): at this point $\widehat{\chi}_\pi$ intersects $\overline{\chi}_\pi$. Otherwise, determinacy is always ensured for $\psi < \frac{\eta-\sigma}{2\sigma}$ (≈ -0.34), as long as $\chi_\pi > \overline{\chi}_\pi = 1$. Between these thresholds we determine two constraints that prevent the central bank from responding too strongly to inflation, namely $\widehat{\chi}_\pi$ and $\widetilde{\chi}_\pi$: the latter lies beyond the numerical range we consider in Figure 1,[10] whereas $\widehat{\chi}_\pi$ kicks in for $\psi \gtrsim 0.45$ in the subspace examined. Such a situation has not been documented by previous studies, which only account for $\psi \leq 0$: in this case a moderate response to inflation always ensures determinacy, as long as the Taylor principle is respected. In contrast, for $\psi > 0$ we need to explore the possibility of introducing additional targets into the policy maker's reaction function.

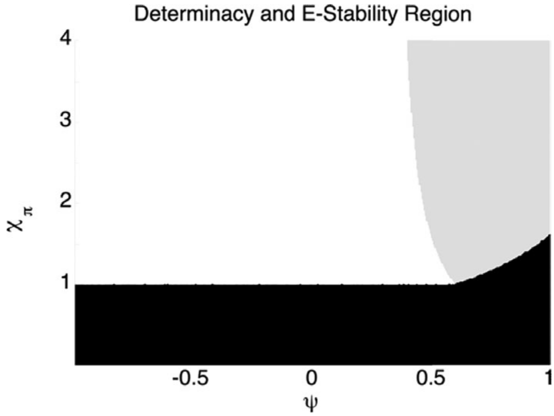

FIGURE 1. Determinacy and E-stability under $r_t = \chi_\pi \pi_t$. Black: indeterminacy and E-instability; light gray: indeterminacy and E-stability; white: determinacy and E-stability.

The next step in the analysis consists of allowing for a joint response to contemporaneous inflation and the output gap: $r_t = \chi_\pi \pi_t + \chi_y y_t$. The analysis in the remainder of the paper will often be complemented by numerical simulations of the model over a parameter subspace of the policy reaction coefficients. Unless otherwise indicated, each numerical exercise will be performed under three different values of the pass-through parameter, $\psi = \{-1, 0, 0.5\}$, to appreciate the effects induced by varying intensities through which the cost channel operates.

As displayed by Figure 2(a), ruling out the cost channel returns the condition embodied by the well-known Taylor principle, by which determinacy is attained as long as $[\kappa(\sigma + \eta)]^{-1}(1 - \beta)\chi_y + \chi_\pi > 1$. However, when cost-side effects are at work, this principle may no longer be sufficient to ensure determinacy. Under a perfect degree of pass-through, the area of indeterminacy expands [see Figure 2(b)]: the minimum bound to inflation responses shifts up along the χ_π axis and increases in χ_y. This situation is analogous to that analyzed by Surico (2008) and Llosa and Tuesta (2009). As the former first pointed out, higher inflation expectations may become self-fulfilling when the cost channel is at work. In fact, a central bank that assigns a positive response to real activity renders the economy more prone to multiple equilibria, as the output gap may not be "negative enough" to offset inflationary pressures.[11] However, it is possible to show that the analysis of Surico (2008) is only valid as long as movements in the policy rate are not amplified by the banking sector. In Figure 2(c) we show that under a stronger degree of pass-through ($\psi = 0.5$), reacting exclusively to the rate of inflation never ensures equilibrium uniqueness. Equilibrium multiplicity may also be a reason for concern if the central bank reacts too weakly to the output gap. A region of indeterminacy can be detected along the χ_π axis, which stems from the intersection between the upper and lower bounds to inflation responses, as reported in Proposition 1. Interestingly, sunspot equilibria in this region are E-stable. This

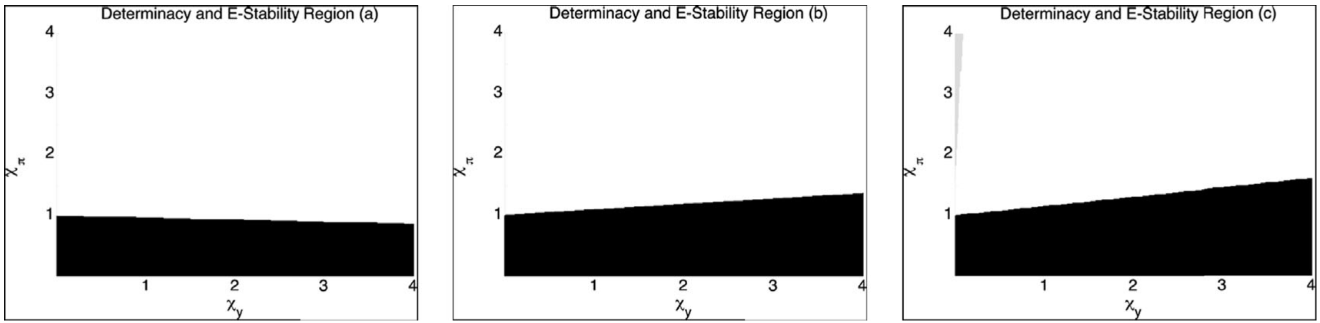

FIGURE 2. Determinacy and E-stability under $r_t = \chi_\pi \pi_t + \chi_y y_t$. ψ is alternatively set to -1 (a), 0 (b), 0.5 (c). Black: indeterminacy and E-instability; light gray: indeterminacy and E-stability; white: determinacy and E-stability.

property has not been documented previously in standard New Keynesian models with contemporaneous data rules, such as those examined by Bullard and Mitra (2002) and Honkapohja and Mitra (2004).[12] Even when accounting for cost-side effects, Llosa and Tuesta (2009) do not point to any discrepancy between the conditions that ensure E-stability and REE uniqueness under contemporaneous data rules [i.e., a situation analogous to that pictured in Figure 2(b)]. However, a disconnection between determinacy and E-stability is highlighted when a positive pass-through is allowed for.[13]

Given the impossibility of attaining a unique equilibrium under strong cost-side effects, the policy maker may need to combine inflation responses with an "appropriate" response to the output gap.[14] To provide some analytical intuition for this result, we find it useful to recall the (sufficient and necessary) conditions for determinacy reported by Llosa and Tuesta (2009), suitably adapted to match our notation:[15]

$$[1 - \beta - (1 + \psi)\,\kappa]\,[\kappa\,(\sigma + \eta)]^{-1}\,\chi_y + \chi_\pi > 1, \tag{8}$$

$$2\sigma\,(1 + \beta) + [1 + \beta + (1 + \psi)\,\kappa]\,\chi_y + \{\eta + \sigma\,[1 - 2\,(1 + \psi)]\}\,\kappa\,\chi_\pi + \kappa\,(\sigma + \eta) > 0. \tag{9}$$

Condition (8) can be interpreted as a generalization of the Taylor principle. This is affected by the cost channel through the output gap response. Specifically, for standard calibrations, the term multiplied by χ_y is negative, because of the presence of $\psi (> -1)$: this argument leads Surico (2008) to conclude that to avoid multiple equilibria the central bank should not respond to output gap movements, as this would weaken the overall response to inflation. However, although condition (8) collapses to $\chi_\pi > 1$ (i.e., the standard Taylor principle, with no role for the cost channel) when $\chi_y = 0$, condition (9) is still affected by ψ. In fact, if we assume $\chi_y = 0$ and $\psi > (\eta - \sigma)(2\sigma)^{-1}$, condition (9) translates into $\chi_\pi < \widetilde{\chi}_\pi$, in agreement with Proposition 1. We should note that $\widetilde{\chi}_\pi$ decreases in ψ, implying that in the event of credit conditions becoming tighter, the maximum response to inflation beyond which determinacy cannot be attained is constrained further.[16] In the limit, credit market distortions may imply that the upper bound becomes so stringent that no determinate outcome is attained (this event occurs whenever $\widetilde{\chi}_\pi$ intersects $\overline{\chi}_\pi = 1$), unless the central bank responds to the output gap, in which case the left-hand terms of both (8) and (9) increase in χ_y.

3.2. Expectations of Current Data in the Policy Function

McCallum (1999) criticizes the use of rules that are not operational, i.e., (i) rules that are expressed in terms of instrumental variables that can hardly be controlled on a high-frequency basis and (ii) rules that require information that cannot plausibly be possessed by the monetary authority. By this definition, contemporaneous data rules such as those explored in Section 3.1 are not operational. In response to this criticism, it is advisable to inspect policy functions based on the expectations of current data. Bullard and Mitra (2002) and Evans and McGough (2005) show that the analysis of determinacy under nowcasting produces results that fully conform to those observed under contemporaneous data rules. This result extends to the case under scrutiny.[17]

However, it is interesting to note that contemporaneous data rules and rules featuring nowcasting have rather different implications in terms of E-stability. In fact, Figure 3(b) shows that under $r_t = \chi_\pi E_t \pi_t + \chi_y E_t y_t$, E-stability may be compromised under a perfect degree of pass-through and even with $\chi_y = 0$. In fact, we note the existence of unique (locally) stationary but E-unstable equilibria.[18] Moreover, the region associated with this type of equilibria enlarges under $\psi = 0.5$, as shown in Figure 3(c). Thus, implementing a rule based on the expectations of current data in the presence of cost-side effects may undermine the possibility of obtaining E-stable equilibria, even when these are unique.

3.3. Forward Expectations in the Policy Function

Figure 4 shows that the region of indeterminacy in the subspace examined is considerably more extended under the forward-looking rule $r_t = \chi_\pi E_t \pi_{t+1} + \chi_y E_t y_{t+1}$ than under the contemporaneous data rule.

To gain some intuition for why forward-looking rules make the system more prone to equilibrium multiplicity, it is useful to re-parameterize the New Keynesian Phillips curve under $r_t = \chi_\pi E_t \pi_{t+1}$:

$$\pi_t = [\beta + \kappa (1 + \psi) \chi_\pi] E_t \pi_{t+1} + \kappa (\sigma + \eta) y_t. \qquad \textbf{(10)}$$

Note that responding to the expected rate of inflation reinforces the feedback from $E_t \pi_{t+1}$ to π_t—thus increasing the chance that expectations of higher inflation become self-fulfilling in the face of inflationary shocks—although leaving the impact of the forcing variable unaffected. Therefore, a shock that raises the nominal rate of interest may generate inflationary pressures that can hardly be offset by the negative output gap, even if $\chi_y = 0$. Importantly, such pressures increase in the degree of pass-through. In contrast, undesirable outcomes are less likely to occur under a contemporaneous data rule ($r_t = \chi_\pi \pi_t$) or one that reacts to contemporaneous expectations of current inflation ($r_t = \chi_\pi E_t \pi_t$), as in these cases χ_π scales the impact of both y_t and $E_t \pi_{t+1}$ on current inflation.

4. ASSET PRICES, THE COST CHANNEL AND DETERMINACY

So far we have shown that allowing for an amplification of movements in the policy rate on bank-lending rates has non-negligible implications for equilibrium dynamics: unlike the case of a less than perfect pass-through, reacting to both inflation and the output gap may be necessary to avoid indeterminacy. This is particularly important when the monetary authority reacts to forward expectations, as in this case the system is more sensitive to feedback effects from expected to current

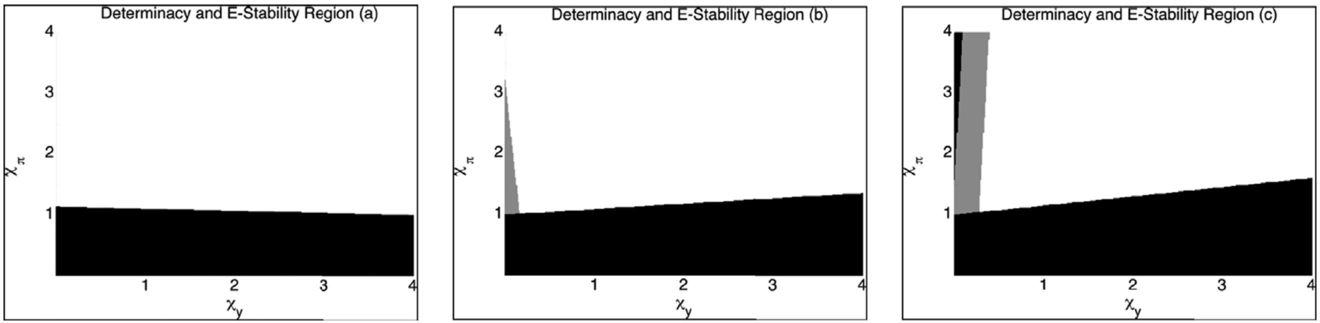

FIGURE 3. Determinacy and E-stability under $r_t = \chi_\pi E_t \pi_t + \chi_y E_t y_t$. ψ is alternatively set to -1 (a), 0 (b), 0.5 (c). Black: indeterminacy and E-instability; dark gray: determinacy and E-instability; white: determinacy and E-stability.

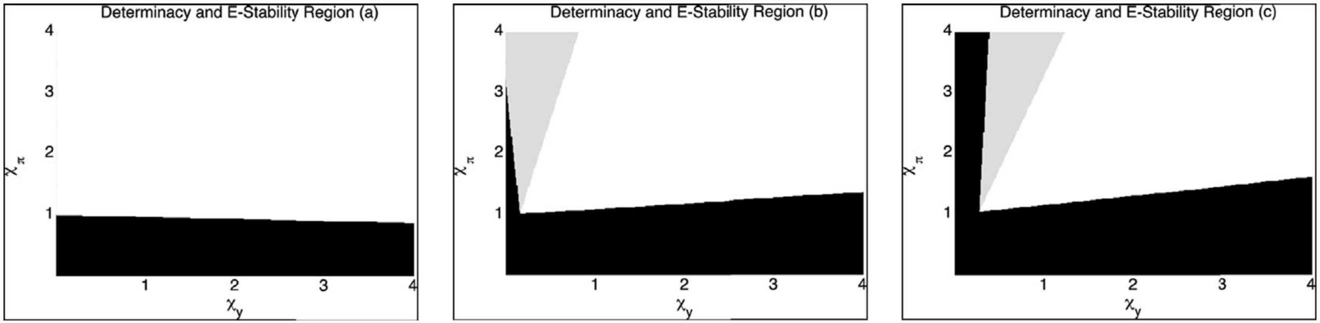

FIGURE 4. Determinacy and E-stability under $r_t = \chi_\pi E_t \pi_{t+1} + \chi_y E_t y_{t+1}$. ψ is alternatively set to -1 (a), 0 (b), 0.5 (c). Black: indeterminacy and E-instability; light gray: indeterminacy and E-stability; white: determinacy and E-stability.

inflation. Concurrently, credit market distortions generally increase the chance to observe learnable sunspots, at least under rules based on contemporaneous data and one-period-ahead expectations, whereas expectations of current data in the Taylor rule make the system more prone to determinate but E-unstable equilibria.

This section highlights important effects emanating from the interplay between credit market distortions and firm profitability. We explore the implications of a central bank that, along with responding to (current or expected) inflation and the output gap, displays some concern for fluctuations in stock prices. We abstract from normative considerations on why the policy maker may want to react to asset prices, merely relying on the evidence that supports this view [see, among others, Rigobon and Sack (2003)].

The general wisdom is that setting the policy rate in response to asset price misalignments renders the system more prone to indeterminacy.[19] Carlstrom and Fuerst (2007) have explored the implications of responding to asset prices for equilibrium determinacy.[20] Their key insight is that in the face of inflationary shocks that lower firm profits (and asset prices) an interest rate rule reacting to stock prices reduces the overall interest rate response to inflation. If the share price response is large enough, indeterminacy cannot be avoided, as the Taylor principle is violated. It is important to stress that Carlstrom and Fuerst (2007) consider a standard situation in which responding to asset prices only affects the lower bound to inflation responses through the conventional demand channel of the monetary transmission mechanism. However, we have noted at different stages of the analysis that in the presence of relevant cost-side effects the upper constraint may represent a reason of concern.

Let us consider the following rule with one-period-ahead expectations:[21]

$$r_t = \chi_\pi E_t \pi_{t+1} + \chi_y E_t y_{t+1} + \chi_q E_t q_{t+1}. \tag{11}$$

Figure 5 graphs the conditions that ensure determinacy over the subspace $\{\chi_\pi, \chi_y\}$ and for $\psi = \{-1, 0, 0.5\}$. In each panel we consider different values of χ_q. Figure 5(a) accounts for the situation examined by Carlstrom and Fuerst (2007) and clearly shows that there are no benefits from reacting to asset prices, as the area of indeterminacy expands as χ_q increases. In the absence of cost-side effects, responding to firm profitability has no implications other than decreasing the possibility of attaining a unique equilibrium. Otherwise, when the cost channel is accounted for, the policy maker needs to select combinations of χ_π and χ_y that fall in the set of determinate equilibria between the lower and the upper constraints on inflation responses. Section 3 has shown that the upper frontier may become an issue of concern for high values of the pass-through coefficient. Figures 5(b) and 5(c) clearly show that a positive reaction to asset prices raises both the lower and the upper bound. However, although increasing χ_q only exerts a negligible impact on the bottom frontier, shifts in the upper bound are far more important. In fact, even a modest response to asset prices prevents the two frontiers from intersecting.

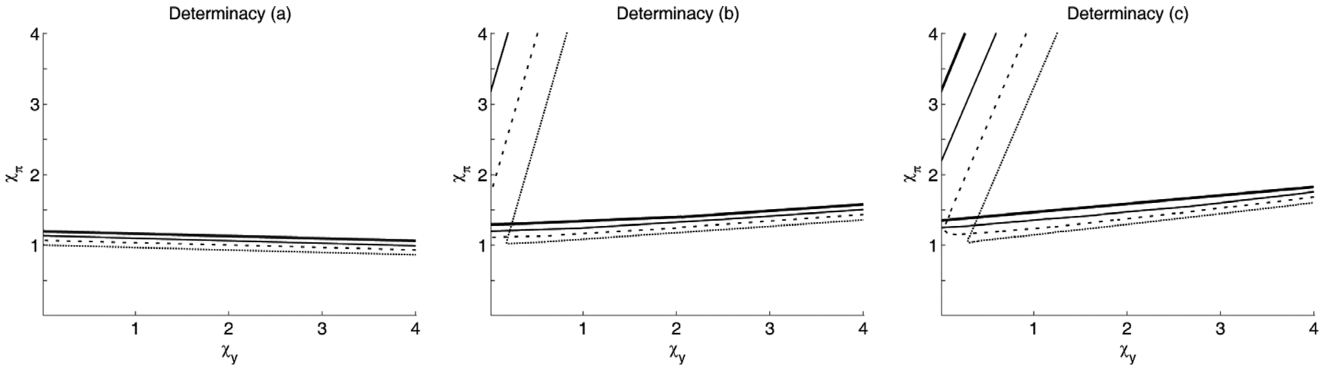

FIGURE 5. Determinacy under $r_t = \chi_\pi E_t \pi_{t+1} + \chi_y E_t y_{t+1} + \chi_q E_t q_{t+1}$. ψ is alternatively set to -1 (a), 0 (b), 0.5 (c). In each panel, χ_q is alternatively set to 0 (dotted line), 0.05 (dashed line), 0.1 (thin continuous line), and 0.15 (thick continuous line).

To provide an intuition for why the lower bound shifts upward, it is useful to explore the effect induced by an inflationary shock. In the model with the cost channel, each percentage point of permanently higher inflation implies a permanent change in the dividend gap of[22]

$$\frac{dd}{d\pi} = -\frac{1+\psi}{\sigma+\eta} - \frac{(1-\beta)\left[(\sigma+\eta)(\theta-1)-1\right]}{\kappa(\sigma+\eta)}, \tag{12}$$

which can be shown to be negative for a wide range of parameter values and to increase in ψ (in absolute value). Thus, as predicted by Carlstrom and Fuerst (2007), a trade-off between inflation and asset price stabilization arises in the effort to attain a unique REE. Note that an analogous trade-off also emerges between inflation and output stabilization, as hinted by (8). In fact, 1% permanent increase in inflation induces a (negative) change in the output gap of $[1-\beta-(1+\psi)\kappa][\kappa(\sigma+\eta)]^{-1}$ percentage points (as opposed to $(1-\beta)[\kappa(\sigma+\eta)]^{-1}$ with no cost channel):[23] this translates into an upward-sloping lower bound in the $\{\chi_\pi, \chi_y\}$ subspace [see Figures 2(b), 2(c)].

Nonetheless, the cost channel urges us to take account of the upper constraint on χ_π as well. With regard to this, adjusting the rate of interest in response to asset price misalignments has effects similar to those of responding to real activity: a positive χ_q leads to a reduction in aggregate supply that outweighs the borrowing effect on inflation dynamics, thus preventing expectations of higher inflation from becoming self-fulfilling. Importantly, this negative effect is amplified by the direct impact of interest rate changes on firm profits: to see this, recall that the elasticity of the dividend gap to the nominal rate of interest, $\mu = (\theta-1)(1+\psi)$, increases in the intensity of cost-side effects. Overall, this translates into an upward shift in the upper bound to χ_π, so that any intersection with the bottom frontier is avoided.

The main point of departure from the result of Carlstrom and Fuerst (2007) lies in the role of the upper bound to inflation responses and its relevance in the presence of strong cost-side effects. It is true that the lower constraint becomes more stringent as χ_q increases, and more so when the cost-channel is accounted for [to see this, consider the term $-(1+\psi)(\sigma+\eta)^{-1}$ in (12), which is null for $\psi=-1$]. However, when cost-side effects are high enough, there are considerably higher gains from responding to asset prices—at least in terms of increased probability of attaining a determinate equilibrium[24]—in that the upper constraint is relaxed.

In the presence of strong cost-side effects that would otherwise prevent the attainment of REE uniqueness, it is advisable to combine inflation responses with an explicit reaction to the output gap and/or asset prices. This allows the central bank to turn the cost channel to its own advantage, through the direct impact of the nominal rate of interest on aggregate supply. Thus, by exploiting the direct influence of interest rates on firm profitability, an interest rate response to asset prices can be quite effective in helping to ensure a unique rational expectations equilibrium.

5. CONCLUDING REMARKS

We have extended the cost channel framework of Ravenna and Walsh (2006) in two main directions: first, following Chowdhury et al. (2006), we allow the introduction of varying degrees of interest rate changes to affect firms' cost of borrowing; second, we consider the direct influence of credit market distortions on firm profitability and stock price dynamics. The standard conditions ensuring REE uniqueness and E-stability are significantly altered in the presence of strong cost-side effects, i.e., when movements in the policy rate are amplified by the lending rate.

When changes in the policy rate are accelerated by the loan rate, conventional inflation-targeting policies may not be effective in ensuring determinacy (i.e., uniqueness of the rational expectations equilibrium) regardless of the timing of the policy rule and the information set available to the policy maker. In contrast to much of the existing literature we show that, along with reacting to the rate of inflation, it may be necessary to adjust the policy rate in response to movements in real activity. Typically, ensuring determinacy requires the policy rate response to inflation response to lie between an upper and a lower bound, each depending on various structural parameters. Although responding to the output gap makes the lower bound to inflation responses more stringent, it produces greater benefits by relaxing the upper constraint. Along the same lines, we show that when the cost channel matters, the policy maker can enhance its ability to attain determinacy and E-stability by reacting to asset prices as well as inflation and output. As in the case considered by Carlstrom and Fuerst (2007), firm profitability reacts negatively in response to inflationary shocks, and more so in the presence of cost-side effects. In otherwise standard frameworks this effect reduces the overall response to inflation, thus making it more difficult to achieve determinacy when the policy rate reacts to asset prices. However, in our setup, along with inducing this relatively small effect, an interest rate response to asset prices counteracts the borrowing cost effect coming from the cost channel, hence relaxing the upper bound on inflation responses, and ensuring the existence of a range of inflation responses that achieve determinacy.

NOTES

1. The literature on the cost channel has shown that, along with the usual demand-side transmission channel, monetary policy significantly affects the supply side of the economy through the influence of the nominal rate of interest on firms' costs of production. See, among others, Christiano et al. (1997), Barth and Ramey (2000), Chowdhury et al. (2006), Ravenna and Walsh (2006) and Tillmann (2008).

2. As discussed by Stiglitz and Weiss (1981), this situation can be rationalized, in the presence of asymmetric information, by the willingness of a firm to invest in risky projects at high levels of the risk-free rate.

3. Variables without a time subscript are evaluated in their steady state. For a generic variable X_t, we denote by $\widehat{x}_t \equiv (X_t - X)X^{-1}$ $[\widehat{x}_t^f \equiv (X_t^f - X)X^{-1}]$ the percentage deviation of its value under sticky (flexible) prices from the steady state level. Moreover, log-linear "gap variables" are reported without superscripts, i.e., $x_t \equiv \widehat{x}_t - \widehat{x}_t^f$.

4. According to their evidence, financial intermediaries acting in imperfectly competitive environments do not fully adapt to changes in the policy rate.

5. See Steinsson (2003) and Ireland (2004).

6. As explained by Carlstrom and Fuerst (2007), other things being equal, an increase in the rate of inflation determines a higher marginal cost and thus lower dividends and share prices, so that the overall response to inflation falls as the response to asset prices increases.

7. Moreover, it is interesting to note that an increase in the elasticity of substitution, θ, exerts a detrimental effect on the asset price gap along two directions: (i) via the output gap (y_t) and (ii) via the interest rate gap (r_t). The second effect is further amplified in the presence of strong cost-side effects ($\psi \gg 0$).

8. The conditions ensuring E-stability under $r_t = \chi_\pi \pi_t$, as well as the determinacy properties of the model economy under alternative rules, are reported in Pfajfar and Santoro (2012).

9. In Pfajfar and Santoro (2012) we also produce a numerical analysis under the parameterization proposed by Woodford (1999). It is important to stress that the evidence reported in the remainder of the paper is not qualitatively affected by alternative parameterizations.

10. It can be shown that the locus $\widetilde{\chi}_\pi$ crosses $\widehat{\chi}_\pi$ from below. Therefore, under the parameterization we use, $\widetilde{\chi}_\pi$ is the relevant upper constraint to inflation responses for $\psi \in [-0.34, 0.33)$.

11. To provide an intuitive explanation of this statement, consider a situation in which the interest rate increases in response to a supply or demand shock, producing a positive ex ante real interest rate. By responding to current inflation, the central bank may trigger even stronger inflationary pressures through the direct impact of the nominal rate of interest on aggregate supply. In this case, the negative output gap induced by the monetary tightening may offset the inflationary pressures arising from the shock. However, an explicit reaction to the output gap may weaken this counterbalancing force, thus rendering the system more prone to indeterminacy.

12. Honkapohja and Mitra (2004) point out that E-stable sunspots may only occur when agents form expectations at time t for time $t+1$, while just observing realizations of the endogenous state variables at period $t-1$. In this respect, sunspots that take the form of a martingale difference sequence are always E-unstable. In contrast, sunspots that take the form of a finite state Markov process may be E-stable for some parameterizations of the policy rule. In addition to finite state Markov sunspots, Evans and McGough (2005) show that in the plausible range of responses to the intermediate targets, E-stable sunspots assuming a common factor representation may be detected. As such, these sunspots represent a threat to monetary policy, as the public could potentially coordinate on them.

13. Pfajfar and Santoro (2012) provide a detailed analytical intuition for this result.

14. This principle gains further relevance under a forward-looking expectational rule, as will be shown in Section 3.3.

15. The conditions reported by Llosa and Tuesta (2009) extend those of Surico (2008) in that they consider no interest rate smoothing and a perfect pass-through between policy and bank-lending rates.

16. Analogous considerations can be extended to the case of $\psi > (\eta/\sigma)$. In fact, also $\overline{\chi}_\pi$ decreases in ψ.

17. In addition, it is possible to prove that the conditions for E-stability under nowcasting are equivalent to those under the forward-looking rule, as Section 3.3 will show.

18. Moreover, as in Evans and McGough (2005), no indeterminate but E-stable equilibria are detected.

19. A longstanding debate concerning the role and scope of central banks in stabilizing asset prices has developed since the contributions of Bernanke and Gertler (1999, 2001) and Genberg et al. (2000). In connection with problems of dynamic stability induced by Taylor rules that respond to share prices, refer to Bullard and Schaling (2002) and Carlstrom and Fuerst (2007). More recently, Pfajfar and Santoro (2011) have shown that adjusting the policy rate in response to asset price growth does not harm dynamic stability and may promote determinacy by inducing interest-rate inertia.

20. We should stress that Carlstrom and Fuerst (2007) explore this situation in the presence of wage rigidity and different timings for money demand. In our setting, such extensions are bound to be of marginal importance. In fact, unlike the sticky price model, in a sticky wage model profits will fall with positive interest rate innovations. Thus, sticky wages induce an effect on firm profits that works in the

same direction as the cost channel. Concurrently, under typical calibrations according to which wages and prices are rather sticky, the money demand timing is almost irrelevant to the stability properties of the New Keynesian model explored by Carlstrom and Fuerst (2007).

21. It is possible to show that analogous principles apply to contemporaneous data rules or under nowcasting.

22. This elasticity can be retrieved by setting, for a generic variable x_t, $E_t x_{t+1} = x_t = x$.

23. Whereas in the baseline scenario (i.e., under $\psi = -1$) any increment in the steady state rate of inflation leads to a higher output gap ($dy/d\pi > 0$), under the cost channel we may assist a permanent reduction in the output gap ($dy/d\pi < 0$), with the magnitude of this response increasing (in absolute value) in ψ.

24. It could be noted that reacting to the stock price gap requires knowledge of asset prices under flexible goods prices. These are typically unobservable. However, it is important to stress that the conditions for determinacy and E-stability would not be qualitatively affected even if we were to consider a linearized sticky price model with variables expressed as percentage deviations from their steady state levels.

25. This condition holds under different plausible parameterizations and is always satisfied under the parameterization we consider in our study.

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APPENDIX: PROOF OF PROPOSITION 1

Under $r_t = \chi_\pi \pi_t$ the NK Phillips curve and the IS curve constitute an autonomous system in which the matrix of structural parameters associated with the forward looking vector is the cofactor:

$$\mathbf{J}_{c33} = \begin{bmatrix} \dfrac{\sigma\beta + \kappa\sigma + \kappa\eta}{\sigma + \kappa\chi_\pi(\eta - \sigma\psi)} & \dfrac{\kappa\sigma^2 + \kappa\eta\sigma}{\sigma + \kappa\chi_\pi(\eta - \sigma\psi)} \\ \dfrac{1 - \chi_\pi(\kappa(1+\psi) + \beta)}{\sigma + \kappa\chi_\pi(\eta - \sigma\psi)} & \dfrac{\sigma - \kappa\sigma\chi_\pi(1+\psi)}{\sigma + \kappa\chi_\pi(\eta - \sigma\psi)} \end{bmatrix}. \tag{A.1}$$

The necessary and sufficient conditions ensuring determinacy are as follows: $|B_c| < 1$ and $|A_c| < 1 + B_c$, where A_c and B_c are the coefficients of the characteristic polynomial of $\mathbf{J}_{c33}$

(i.e., $\lambda^2 + A_c\lambda + B_c = 0$):

$$B_c \equiv \frac{\beta\sigma}{\sigma + \kappa\chi_\pi(\eta - \sigma\psi)}, \tag{A.2}$$

$$A_c \equiv \frac{\kappa\sigma\chi_\pi(1+\psi) - \sigma(1+\beta) - \kappa(\sigma+\eta)}{\sigma + \kappa\chi_\pi(\eta - \sigma\psi)}. \tag{A.3}$$

Let us first focus on $|B_c| < 1$, which translates into

$$\frac{\beta\sigma}{\sigma + \kappa\chi_\pi(\eta - \sigma\psi)} < 1 \tag{A.4}$$

and

$$\frac{\beta\sigma}{\sigma + \kappa\chi_\pi(\eta - \sigma\psi)} > -1. \tag{A.5}$$

We start by manipulating (A.4), multiplying both sides by $\sigma + \kappa\chi_\pi(\eta - \sigma\psi)$: this term is always positive for $\psi < \frac{\eta}{\sigma}$. In contrast, when $\psi > \frac{\eta}{\sigma}$, we need to introduce a restriction on κ to ensure that $\sigma + \kappa\chi_\pi(\eta - \sigma\psi)$ is positive, namely $0 < \kappa < \frac{\sigma}{\chi_\pi(\sigma\psi - \eta)}$.[25] To derive an explicit condition for χ_π we divide both sides of the resulting inequality by $\kappa(\eta - \sigma\psi)$. This term is negative for $\sigma\psi > \eta$: in this case we end up with $\chi_\pi < \frac{\sigma(\beta-1)}{\kappa(\eta-\sigma\psi)}$. Otherwise, when $\psi < \frac{\eta}{\sigma}$ we obtain $\chi_\pi > \frac{\sigma(\beta-1)}{\kappa(\eta-\sigma\psi)}$. Note that the term on the RHS of the last inequality is always negative.

We now consider (A.5). Again, to isolate χ_π on the LHS we need to divide both sides of the inequality by $\kappa(\eta - \sigma\psi)$. Thus, if $\psi > \frac{\eta}{\sigma}$, we obtain

$$\chi_\pi < \frac{\sigma(\beta+1)}{\kappa(\sigma\psi - \eta)}. \tag{A.6}$$

When $\psi > \frac{\eta}{\sigma}$, the term $\frac{\sigma(\beta+1)}{\kappa(\sigma\psi-\eta)}$ is always positive under the restriction characterizing the baseline parameterization. In the alternative case (i.e., $\psi < \frac{\eta}{\sigma}$), we obtain $\chi_\pi > \frac{\sigma(\beta+1)}{\kappa(\sigma\psi-\eta)}$: as the term on the RHS of the last inequality is always negative, this condition is nested in $\chi_\pi > 1$.

Finally, we turn our attention to the second condition for determinacy, $|A_c| < 1 + B_c$, which translates into

$$\frac{\kappa\sigma\chi_\pi(1+\psi) - \sigma(1+\beta) - \kappa(\sigma+\eta)}{\sigma + \kappa\chi_\pi(\eta - \sigma\psi)} < 1 + \frac{\beta\sigma}{\sigma + \kappa\chi_\pi(\eta - \sigma\psi)}, \tag{A.7}$$

$$\frac{\kappa\sigma\chi_\pi(1+\psi) - \sigma(1+\beta) - \kappa(\sigma+\eta)}{\sigma + \kappa\chi_\pi(\eta - \sigma\psi)} > -1 - \frac{\beta\sigma}{\sigma + \kappa\chi_\pi(\eta - \sigma\psi)}. \tag{A.8}$$

Once again, we assume that the restriction on κ holds true: this allows us to write (A.7) as $\chi_\pi\kappa[\sigma(1+2\psi) - \eta] < 2\sigma(1+\beta) + \kappa(\sigma+\eta)$. Thus, we have to evaluate the sign of $\kappa[\sigma(1+2\psi) - \eta]$: this turns out to be always positive iff $\psi < \frac{\eta}{\sigma}$. Otherwise, for $\psi \gtrless \frac{\eta-\sigma}{2\sigma}$ the relevant conditions are $\chi_\pi \lessgtr \frac{2\sigma(1+\beta)+\kappa(\sigma+\eta)}{\kappa(\sigma(1+2\psi)-\eta)}$. However, for $\psi < \frac{\eta-\sigma}{2\sigma}$, the term $\frac{2\sigma(1+\beta)+\kappa(\sigma+\eta)}{\kappa(\sigma(1+2\psi)-\eta)}$ is negative, so that the resulting condition is nested in $\chi_\pi > 1$.

Finally, we consider (A.8). Algebraic manipulations similar to those followed for (A.7) show that the only relevant condition for determinacy is $\chi_\pi > 1$. Q.E.D.