**STELLA MARIS COLLEGE(AUTONOMOUS), CHENNAI-86.**

(For candidates admitted during the year 2011-12)

**M.PHIL DEGREE EXAMINATION - APRIL 2012**

**SUBJECT CODE: 11MT/RO/GT2 05**

**BRANCH I –MATHEMATICS**

**COURSE: OPTIONAL PAPER**

**PAPER : GRAPH THEORY**

**TIME: 3 HOURS** **MAX MARKS:100**

**ANSWER ANY FIVE:** **(5 x 20 = 100)**

1. a) Prove that a graph is bipartite if and only if it has no odd cycle.
   b) Which of the following are graphic sequences?
   i) (5,5,,4,3,2,2,2,1) ii) (5,5,4,4,2,2,1,1) iii) (5,5,5,3,2,2,1,1) iv) (5,5,5,4,2,1,1,1)
   c) Prove that an edge is a cut-edge if and only if it belongs to no cycle. (7 + 8 + 5)

2. a) Prove that for an $n$-vertex graph $G$ (with $n \geq 1$) the following are equivqlent:
   i) $G$ is connected and has no cycles.
   ii) $G$ is connected and has $n - 1$ edges.
   iii) $G$ has $n$ -1 edges and has no cycles.
   iv) $G$ has no loops and has for each $u, v \in V(G)$, exactlty one $u$-$v$ path.
   b) Find the number of spanning trees of a complete graph on 6 vertices. (12 + 8)

3. a) Define matching and Maximum matching and find the same for $K_{4,3}$.
   b) Determine the minimal size of a maximal matching in a cycle.
   c) State and prove Hall's theorem. (6 + 4 + 10)

4. a) Define chromatic number and find the same for a wheel on 6 vertices and Petersen graph.
   b) State and prove Brook's theorem.
   c) State any two applications of colouring problem. (6 + 10 + 4)

5. a) Define crossing number and obtain the same for $K_{4,2}$.
   b) State and prove Euler's formula.
   c) Prove that if $G$ is a graph with fewest edges among all nonplanar graphs without Kuratowski subgraphs, then $G$ is 3-connected. (5 + 7 + 8)

6. a) State and prove the necessary and sufficient condition for a graph to be Eulerian.
   b) State and prove Dirac's condition for a Hamiltonian graph. (10 + 10)

7. a) Define an Interconnection Network.
   b) Explain the basic principles of Network Design. (4 +16)

8. a) Explain the various methods for constructing topological structure of Interconnection Network.
   b) Draw an Hypercube and explain it's properties. (10+10)