STELLA MARIS COLLEGE (AUTONOMOUS), CHENNAI-600 086

M.Phil. DEGREE: BRANCH I – MATHEMATICS

(Effective from the academic year 2011–2012)

SUBJECT CODE: 11MT/RO/FA205

END SEMESTER EXAMINATION APRIL 2012

TITLE: FUNCTIONAL ANALYSIS
TIME: 3 HOURS

MAX: 100 MARKS

Answer any five questions

5×20=100

1. Prove the completeness of l_p and L_p.
2. State and prove Principle of Contraction Mapping.
3. State and prove Arzela's theorem.
4. State and prove the spectral properties of bounded self adjoint operators.
5. Prove that every positive bounded self adjoint linear operator on a complex Hilbert space has a positive square root which is unique and commutes with every bounded linear operator.
6. In complete space E let $x(t)$ be a continuous function with $t \in [a,b]$ and let A_{τ_n} be a sequence of partitions of the interval $[0,1]$ such that $\tau_n \to 0$ as $n \to \infty$. Prove that if we form the integral sums $S[A_{\tau_n}, x(t)]$, then they tend to a limit $S = \lim_{n\to\infty} S[A_{\tau_n}, x(t)]$ as $n \to \infty$, where the limit S does not depend on the choice of A_{τ_n}.
7. a) Define Extremal point.

   b) Prove that Haar condition is sufficient for the uniqueness of the best approximation.

   c) State and prove the Haar uniqueness theorem.
8. State and prove the necessary and sufficient condition for metrization.