STELLA MARIS COLLEGE (AUTONOMOUS), CHENNAI – 600 086

M. Phil. DEGREE EXAMINATION APRIL 2012

DEPARTMENT OF MATHEMATICS

ADVANCED ALGEBRA

CLASS: M.PHIL. **MAX: 100 MARKS**

CODE:11MT/RO/AA2 05 **TIME: 3 HOURS**

Answer any Five questions (Each question carries 20 marks)

1. State and prove Jacobson's density theorem on semi simple R- modules.
2. Prove the structure theorem for semi simple Rings.
3. i) Prove that an exact functor from $\mathcal{C}$ to $\mathcal{D}$ transforms every exact sequence in $\mathcal{C}$ to an exact sequence in $\mathcal{D}$.

   ii) Define Projective module and free module. Prove that every free module is projective.
4. i) State and prove Schanuel's lemma.

   ii) State and prove the equivalent conditions for a module to be projective.
5. a) Define Injective module and give an example.

   b) State and prove Baer's Theorem.

   c) Prove that every Z-module can be embedded in an injective Z-module.
6. a) Prove that every finitely generated projective A-module P is finitely presented.

   b) Prove that every projective module of rank n over a semi-local ring is free of rank n.
7. State and prove Wedderburn's Structure theorem on semi-simple R – algebras
8. i) Define a fuzzy set and a fuzzy subgroup.

   ii) Prove that μ is a fuzzy subgroup of G if and only if μ_a is a subgroup of G.

   iii) Define a normal fuzzy subgroup . If $\mu \in N(F(G))$ and $\vartheta \in F(G)$, prove that $\mu \cap \vartheta$ is a normal fuzzy subgroup of ϑ.
