

STELLA MARIS COLLEGE (AUTONOMOUS), CHENNAI – 600 086

(Effective from the academic year 2011-2012)

CODE:11MT/RO/AA2 05

M. Phil. DEGREE EXAMINATION APRIL 2014

BRANCH I – MATHEMATICS

ADVANCED ALGEBRA (OPTIONAL)

CLASS: M.PHIL.

MAX: 100 MARKS

TIME: 3 HOURS

Answer any Five questions (Each question carries 20 marks)

1. i) Prove that every submodule and every factor module of a semisimple module is semisimple.
ii) State and prove Jacobson's theorem on semisimplicity.
 2. i) Prove that an algebra is semisimple iff it is right Artinian and $\text{rad } A = 0$.
ii) Define Projective module and free module. Prove that every free module is projective.
 3. i) Define pull back and push out in modules and prove that pull back and push out always exist and unique up to canonical isomorphism.
ii) Prove that every Z -module can be embedded in an injective Z -module.
 4. i) State and prove Schanuel's lemma.
ii) State and prove the equivalent conditions for a module to be projective.
 5. i) Define Injective module and give an example.
ii) State and prove Baer's Theorem.
 6. i) Prove that every finitely generated projective A -module P is finitely presented.
ii) Prove that every projective module of rank n over a semi-local ring is free of rank n .
 7. State and prove Wedderburn's Structure theorem on semi-simple R – algebras.
 8. i) If A is an R - algebra and M is a free right A - module, prove that
$$E_A(\oplus_n M) \cong M_n(E_A(M))$$

ii) State and prove Masche's Theorem.
-