STELLA MARIS COLLEGE(AUTONOMOUS), CHENNAI-86.

(For candidates admitted after the academic year 2011-12)

M.PHIL DEGREE EXAMINATION - APRIL 2014

SUBJECT CODE: 11MT/RO/GT2 05

BRANCH I –MATHEMATICS

COURSE: OPTIONAL PAPER

PAPER : GRAPH THEORY

TIME: 3 HOURS **MAX MARKS:100**

ANSWER ANY FIVE: **(5 x 20 = 100)**

1. a) Prove that a graph is bipartite if and only if it has no odd cycle.

   b) Define a cut-edge and an edge-cut with an example and prove that an edge is a cut-edge if it belongs to no cycle.

   c) Prove that complete graph K_n can be expressed as the union of k bipartite graphs if and only if $n \leq 2^k$. (7 + 8 + 5)

2. a) Prove that for an n-vertex graph G (with $n \geq 1$) the following are equivqlent:

   i) G is connected and has no cycles.

   ii) G is connected and has $n - 1$ edges.

   iii) G has n -1 edges and has no cycles.

   iv) G has no loops and has for each $u, v \in V(G)$, exactlty one u-v path.

   b) Define a degree sequence and determine which of the following sequence is graphic: (5,5,4,3,2,2,2,1); (5,5,4,4,2,2,1,1); (5,5,5,3,2,2,1,1); (5,5,5,4,2,1,1,1) (12 + 8)

3. a) Define perfect matching and Maximum matching and find the same for $K_{3,4}$.

   b) State and prove Hall's theorem.

   c) Prove or disprove that every tree has at most one perfect matching. (6 + 9 + 5)

4. a) Define chromatic number and find the same for a cycle on 13 vertices and P_{11}.

   b) State and prove Brook's theorem.

   c) State any two applications of colouring problem. (6 + 10 + 4)

5. a) Define crossing number and obtain the same for P(6,2).

b) State and prove Euler's formula.

c) Prove that if G is a graph with fewest edges among all nonplanar graphs without Kuratowski subgraphs, then G is 3-connected. (5 + 7 + 8)

6. a) State and prove the necessary and sufficient condition for a graph to be Eulerian.

b) State and prove Dirac's condition for a Hamiltonian graph. (10 + 10)

7. a) Define an Interconnection Network.

b) Explain the basic principles of Network Design. (4 +16)

8. a) Describe the three equivalent ways to define an Interconnection network with an example.

b) Draw a Kautz graph and a circulant graph and explain its properties. (8+12)
