STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2009 – 10 & thereafter)

SUBJECT CODE : MT/PE/SP33

M. Sc. DEGREE EXAMINATION, NOVEMBER 2011
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : ELECTIVE
PAPER : STOCHASTIC PROCESSES
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 X 8 = 40)**
ANSWER ANY FIVE QUESTIONS

1. Derive the density function of S_n, the arrival of the n^{th} event.
2. Given $N(t) = n$. Show that the n arrival times $S_1, S_2, \cdots, S_n$ have the same distribution as the order statistics corresponding to n independent random variables uniformly distributed on the interval $(0, t)$.
3. Show that the output process of $M/G/\infty$ queue is a nonhomogeneous Poisson process having intensity function $\lambda G(t)$.
4. Let X be a random variable having distribution that is independent of $W = \sum_{i=1}^{N} X_i$. For any function $h(x)$, prove that $E[Wh(W)] = \lambda E[Xh(W + X)]$.
5. Show that the renewal function $m(t) = \sum_{n=1}^{\infty} F_n(t)$. Also show that $m(t) < \infty$, for all $0 \le t < \infty$.
6. If $\{S_n, n \ge 1\}$ is a simple random walk, prove that
$$P\{S_n = i \mid |S_n| = i, |S_{n-1}| = i_{n-1}, \cdots, |S_1| = i_1\} = \frac{p^i}{p^i + q^i}.$$
7. In a Yule process with $X(0) = 1$, compute the expected sum of the ages of members of population at time t.

SECTION – B **(3 X 20 = 60)**
ANSWER ANY THREE QUESTIONS

8. Let $\tau_1, \tau_2, \cdots, \tau_n$ denote the oredered values from a set of n independent uniform $(0, t)$ uniform random variables. Let $Y_1, Y_2, \cdots$ be independent and identically distributed random variables that are also independent of $\{\tau_1, \tau_2, \cdots, \tau_n\}$. Prove that
$$P(Y_1 + \cdots + Y_k < \tau_k, k = 1, \cdots, n \mid Y_1 + \cdots + Y_n = y) = 1 - \frac{y}{t}, o < y < t$$
$$= 0 \quad \text{otherwise}$$

9. a) Derive the distribution function for busy period, stating the necessary lemmas.
   b) Let w be a compound Poisson random variable with Poisson parameter $\lambda = 4$ and with $P(X_i = i) = \frac{1}{4}$, $i = 1,2,3,4$. Determine $P(w = 4)$ using the recursion formula.
   $P_0 = e^{-\lambda}$,
   $P_n = \frac{\lambda}{n} \sum_{j=1}^{n} j\alpha_j P_{n-j}$

10. If $E[R] < \infty$ and $E[X] < \infty$, then show that
   (i) with probability 1, $\dfrac{R(t)}{t} \to \dfrac{E[R]}{E[X]}$ as $t \to \infty$
   (ii) $\dfrac{E[R(t)]}{t} \to \dfrac{E[R]}{E[X]}$ as $t \to \infty$

11. Prove that an irreducible a periodic Markov chain belongs to one of the following two cases:
   (i) Either the states are all transient or all null recurrent; in this caw, $P_{ij}^{n} \to 0$ as $n \to \infty$ for all i, j and there exists no stationary distribution.
   (ii) Or else, all states are positive recurrent, that is, $\pi_j = \lim_{n\to\infty} P_{ij}^{n} > 0$. In this case, $\{\pi_j, j = 0,1,2,\cdots\}$ is a stationary distribution and there exists no other stationary distribution.

12. State and prove Kolmogorov's Backward and Forward equations of continuous-time Markov chains.

⅄⅄⅄⅄⅄⅄⅄⅄⅄⅄⅄