STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2009 – 10 & thereafter)

SUBJECT CODE: MT/PC/PD34

M. Sc. DEGREE EXAMINATION, NOVEMBER 2011
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : CORE
PAPER : PARTIAL DIFFERENTIAL EQUATIONS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 X 8 = 40)**
ANSWER ANY FIVE QUESTIONS

1. Form the PDE by eliminating the arbitrary function
   (i) $Z = f(x + ib) + g(x - ib)$ where $i = \sqrt{-1}$
   (ii) $f(x + y + z, x^2 + y^2 + z^2) = 0$

2. Use Jacobi's method to solve $p^2x + q^2y = 8$.

3. If f and g be arbitrary functions of their respective arguments. Show that $u = f(x - vt + iy) + g(x - vt - i\alpha y)$ is a solution of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ provided $\alpha^2 = 1 - \frac{v^2}{c^2}$.

4. Solve the equation $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^3 \partial y} - \frac{\partial^3 y}{\partial x \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}$.

5. Find the solution of Interior Neumann problem for a circle.

6. State and prove uniqueness theorem on heat conduction equation.

7. Derive the one dimensional wave equation.

SECTION – B **(3 X 20 = 60)**
ANSWER ANY THREE QUESTIONS

8. (a) Show that the following PDES $xp - yq = x$ and $x^2p + q = xz$ are compatible and hence find their solution.
   (b) Find the complete integral of $x^2p^2 + y^2q^2 - 4 = 0$ using charpit's method.

9. (a) Find the solution of the equation $\nabla_1^2 z = e^{-x} \cos y$ which tends to zero as $x \to \infty$ and has the value $\cos y$ when $x = 0$.
   (b) Reduce the Tricomi equation, $u_{xx} + xu_{yy} = 0, x \neq 0$ for all x, y to canonical form.

10. (a) Derive Poisson equation.

    (b) Find the solution of interior Dirichlet problem for a circle.

11. (a) Find the solution of the one-dimensional diffusion equation satisfying the boundary conditions.

    (i) T is bounded as $t \to \infty$

    (ii) $\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0$, for all t

    (iii) $\left.\frac{\partial T}{\partial x}\right|_{x=a} = 0$ for all t

    (iv) $T(x, 0) = x(a - x),\ 0 < x < a$.

    (b) State and prove the maximum-minimum principle.

12. (a) State and prove D'Alemberts solution of one-dimensional wave equation.

    (b) By separating variables, show that one dimensional wave equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ has a solution of the form A exp(± inx ±inct), where A and n are constants. Hence show that functions of the form

    $z(x, t) = \sum_r (A_r \cos\frac{r\pi ct}{a} + B_r \sin\frac{r\pi ct}{a})\, sin\left(\frac{r\pi x}{a}\right)$ where Ar's and Br's are constants, satisfy the wave equation and the boundary conditions

    $z(0, t) = 0,\ z(a, t) = 0,\ \forall t.$

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