STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2009 – 10 & thereafter)

SUBJECT CODE: MT/PC/MS34

M. Sc. DEGREE EXAMINATION, NOVEMBER 2011
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : CORE
PAPER : MATHEMATICAL STATISTICS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 X 8 = 40)**
ANSWER ANY FIVE QUESTIONS

1. Find the characteristic function of a normal distribution and hence obtain its mean and variance.
2. Prove that X has a one-point distribution if and only if its variance is zero.
3. Let $F_n(x) \quad (n = 1,2,\dots)$ be distribution function of the random variable X_n. Prove the necessary and sufficient condition for the stochastic convergence of the sequence $\{X_n\}$.
4. State and prove Poisson law of large numbers.
5. Define chi-square distribution. Derive the distribution function. Write its mean and variance.
6. Prove that S^2 is not unbiased estimator of variance σ^2 of random variable X and ${S_1}^2$ is an unbiased estimator of σ^2.
7. If a random variable has Poisson distribution with unknown parameter λfind the estimate of λ using method of maximum likelihood.

SECTION – B **(3 X 20 = 60)**
ANSWER ANY THREE QUESTIONS

8. a) If X_1 and X_2 are independent Poisson random variables, find the characteristic function of $X_1 - X_2$.

   b) If the characteristic function of the random variable is $\varphi(t) = \exp\left(-t^2/2\right)$.

   Find its density function.
9. Define Cauchy distribution. Obtain its characteristic function. Also check the validity of addition theorem for Cauchy random variable.
10. State and prove Levy-Cramer Theorem.
11. Derive Fisher's Z distribution.
12. State and prove Cramer-Rao inequality.

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