STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2009 – 10 & thereafter)

SUBJECT CODE : MT/PC/CM34

M. Sc. DEGREE EXAMINATION, NOVEMBER 2011
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : CORE
PAPER : CONTINUUM MECHANICS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 X 8 = 40)**
ANSWER ANY FIVE QUESTIONS

1. If the stress sensor $\sigma_{ij} = \begin{pmatrix} \sigma & a\sigma & b\sigma \\ a\sigma & \sigma & c\sigma \\ b\sigma & c\sigma & \sigma \end{pmatrix}$, determine the constants a,b,c so that the stress vector on the octahedral plane $\left(\hat{n} = \frac{1}{\sqrt{3}}\hat{e}_1 + \frac{1}{\sqrt{3}}\hat{e}_2 + \frac{1}{\sqrt{3}}\hat{e}_3 \right)$ vanishes.
2. Determine the principal deviator stress values for the stress tensor
$$\sigma_{ij} = \begin{pmatrix} 10 & -6 & 0 \\ -6 & 10 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
3. The displacement field of continuum body is given by $x_1 = X_1,\ x_2 = X_2 + AX_3,$ $x_3 = x_3 + AX_2$, where A is constant. Determine the displacement vector components in both the material and spatial form.
4. A velocity field is given by $v_1 = x_1/(1+t), v_2 = 2x_2/(1+t), v_3 = 3x_3/(1+t)$ .Obtain the displacement relations $x_i = x_i(X,t)$ and determine the acceleration components in Lagrangian form for the motion.
5. For the steady velocity field $v = 3{x_1}^2 x_2 \hat{e}_1 + 2{x_2}^2 x_3 \hat{e}_2 + 3x_1 x_2 {x_3}^2 x_2 \hat{e}_3$, determine the rate of extension at $P(1,1,1)$ in the direction of $v = \frac{1}{5}(3\hat{e}_1 - 4\hat{e}_3)$.
6. (a) Show that the velocity field $v_i = Ax_i / r^3$ where $x_i x_i = r^2$ and A is an arbitary constant , satisfies the continuity equation for an incompressible flow.
   (b) For the velocity field $v_i = x_i/(1+t)$ show that $\rho x_1 x_2 x_3 = \rho_0 X_1 X_2 X_3$.
7. Express the engineering constants ν and E in terms of the Lame' constants λ and μ .

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SECTION – B **(3 X 20 = 60)**

ANSWER ANY THREE QUESTIONS

8. (a) Establish the relation between the stress tensor and the stress vector.

(b) Evaluate the stress invariants for the stress tensor $\sigma_{ij} = \begin{pmatrix} 6 & -3 & 0 \\ -3 & 6 & 0 \\ 0 & 0 & 8 \end{pmatrix}$.

9. (a) A displacement field is given by $= X_1 X_3{}^2 \hat{e}_1 + X_1{}^2 X_2 \hat{e}_2 + X_2{}^2 X_3 \hat{e}_3$. Determine independently the material deformation gradient **F** and the material displacement gradient **J** and verify **J=F-I.**

(b) A continuum body undergoes the deformation $x_1 = X_1, x_2 = X_2 + AX_3, x_3 = X_3 + AX_2$ where A is constant . Compute the deformation tensor **G** and use this to determine the Lagrangian finite strain tensor $\boldsymbol{L_G}$.

10. (a) A displacement field is specified by $\boldsymbol{u} = X_1{}^2 X_2 \hat{e}_1 + (X_2 - X_3{}^2)\hat{e}_2 + X_2{}^2 X_3 \hat{e}_3$.

Determine the relative displacement vector $d\boldsymbol{u}$ in the direction of the $-X_2$ axis at $P(1,2,-1)$.Detemine the relative displacements $\boldsymbol{u}_{q_i} - \boldsymbol{u}_p$ for $Q_1(1,1,-1), Q_2(1, {}^3/_2, -1), Q_3(1, {}^7/_4, -1)$ and $Q_4(1, {}^{15}/_8, -1)$ and compare their direction with the direction of $d\boldsymbol{u}$**.**

(b) Explain the terms: Material derivative , Local rate of change ,convective rate of change and Material derivative operator.

11. (a) A steady velocity field is given by $\vec{v} = (x_1{}^3 - x_1 x_2{}^2)\hat{e}_1 + (x_1{}^2 x_2 - x_2)\hat{e}_2$. Determine the unit relative velocity with respect to $P(1,1,3)$ of the particles at $Q_1(1,0,3), Q_2\left(1, \frac{3}{4}, 3\right), Q_3\left(1, \frac{7}{8}, 3\right)$.

(b) Using linear momentum principle, obtain the equations of motion of a moving continuum.

12. Obtain the generalised Hooke's law for an isotropic body in terms of the elastic constants λ *and* μ .Show that the strain energy density function u^* for an isotropic Hookean solid may be expressed in terms of the strain tensor by $u^* = \frac{\lambda}{2}\varepsilon_{ii}\varepsilon_{jj} + \mu\varepsilon_{ij}\varepsilon_{ij}$.

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