STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2011 – 12)

SUBJECT CODE : 11MT/PC/RA14

M. Sc. DEGREE EXAMINATION, NOVEMBER 2011
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : CORE
PAPER : REAL ANALYSIS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 X 2 = 10)**
ANSWER ALL QUESTIONS

1. Define (i) an interior point (ii) a metric space
2. Define (i) Cesaro Summability of a series (ii) infinite product
3. Justify the statement "the operations of limit and integration cannot always be interchanged" with a suitable example.
4. Let $f: R^2 \rightarrow R^3$ be defined by the equation $f(x, y) = (sinxcosy, sinxsiny, cosxcosy)$, determine the Jacobian matrix $Df(x, y)$.
5. Find and classify the extreme values of the function $f(x, y) = y^2 - x^3$.

SECTION – B **(5 X 6 = 30)**
ANSWER ANY FIVE QUESTIONS

6. Prove that (i) The union of any collection of open sets is an open set.
   (ii) The intersection of a finite collection of open sets is open.
7. Assume that each $a_n \geq 0$, the prove that the product $\prod(1 - a_n)$ converges if and only if $\sum a_n$ converges.
8. State and prove the Cauchy condition for uniform convergence of sequence of functions.
9. State and prove the chain rule to compute the total derivative of $\boldsymbol{h}$ in terms of total derivative of $\boldsymbol{f}$ and of $\boldsymbol{g}$.
10. State and prove the Mean-value theorem for differentiable functions on $\mathbb{R}^n$.
11. Assume that $\boldsymbol{f} = (f_1, f_2, .., f_n)$ has continuous partial derivatives $D_j f_i$ on an open set S in R^n, and that the Jacobian determinant $\boldsymbol{J_f(a)} \neq 0$ for some point $\boldsymbol{a}$ in S, the prove that there is an n-ball $B(\boldsymbol{a})$ on which $\boldsymbol{f}$ is one-to-one.
12. State and prove the second derivative test for extrema.

SECTION – C **(3 X 20 = 60)**

ANSWER ANY THREE QUESTIONS

13. a) State and prove the Cantor intersection theorem.

    b) Let S be a subset of R^n, then show that the following three statements are equivalent :

    (i) S is compact

    (ii) S is closed and bounded

    (iii) Every infinite subset of S has an accumulation point in S.

14. a) State and prove the Rearrangement theorem for double series.

    b) Determine whether the infinite product $\prod_{n=2}^{\infty} \frac{n^3-1}{n^3+1}$ converges or not.

15. a) Assume that $f_n \to f$ uniformly on $S \subseteq \mathbb{R}^1$. If each f_n is continuous at a point c of S then

    show that the limit function f is also continuous at c.

    b) State and prove Bernstein's theorem.

16. a) Justify the statement that "a function can have a finite directional derivative $\boldsymbol{f}'(\boldsymbol{c}, \boldsymbol{u})$ for every $\boldsymbol{u}$ but may fail to be continuous at $\boldsymbol{c}$" with a suitable example.

    b) If both partial derivatives $D_r\boldsymbol{f}$ and $D_k\boldsymbol{f}$ exist in an n-ball $B(\boldsymbol{c};\delta)$ and if both are differentiable at c, then prove that $D_{r,k}\boldsymbol{f}(\boldsymbol{c}) = D_{k,r}\boldsymbol{f}(\boldsymbol{c})$.

17. State and prove the implicit function theorem.

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