STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2011 – 12)

SUBJECT CODE : 11MT/PC/OD14

M. Sc. DEGREE EXAMINATION, NOVEMBER 2011
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : CORE
PAPER : ORDINARY DIFFERENTIAL EQUATIONS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL THE QUESTIONS: **(5 x 2 = 10)**

1. Define linear dependence and independence.
2. Solve $x''' + 3x'' + 3x' + x = 0$.
3. Define an analytic function.
4. Show that $J_0{}'(x) = -J_1(x)$.
5. Define Green's function.

SECTION – B

ANSWER ANY FIVE QUESTIONS: **(5 x 6 = 30)**

6. Solve $x'' - \frac{2}{t}x' + \frac{2}{t^2}x = t\sin t,\ t \in [1, \infty)$ using the method of variation of parameters.
7. Compute the wronskian of e^x, e^{2x} and 1.
8. Consider the equation $x''' - 4x' = 0$.

   (a) compute three linearly independent solution

   (b) Find the solution when $x(0) = 0, x'(0) = 1$ & $x''0) = 0$.
9. Consider the equation $y'' + y = 0$ find its general solution

   $y = \Sigma a_n x^n$ *in the form* $y = a_0 y_1(x) + a_1 y_2(x)$ where $y_1(x)$ & $y_2(x)$ are power series.
10. If $P_m(t)$ and $P_n(t)$ are Legendre polynomials then prove that

    $\int_{-1}^{1} P_m(x)P_n(x)dx = 0 \quad (m \neq n)$
11. Show that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\ \sin x$
12. State and prove Picard's Boundary Value theorem.

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SECTION – C

ANSWER ANY THREE QUESTIONS : (3 x 20 = 60)

13. (a) Let $x_1(t), x_2(t), \ldots, x_n(t)$ be linearly independent solution of $L(x) = 0$ on the interval I, then any solution of $x(t)$ of $L(x) = 0$ on I is of the form
$x(t) = C_1x_1(t) + C_2x_2(t) + \ldots + C_nx_n(t), t \in I$ *where* $C_1, C_2, \ldots, C_n$ are some constants.

    (b) Solve $t^2x'' - 2x = 2t - 1$.

14. (a) State and prove Abel's formula.

    (b) Find the general solution of the non-homogeneous IVP given by
$L(y) = d(t), y(t_0) = y'(t_0) = 0\ t, t_0 \in I$

15. Show that $Z(t) = \sum_{k-0}^{\infty} a_k t^k$ is a solution of Hermite equation $x'' - 2tx' + 2x = 0$.

16. (a) If P_n is a Legendre polynomial then prove that $\int_{-1}^{1} {P_n}^2(t)dt = \frac{2}{2n+1}$

    (b) Show that the Bessel function of the first kind of order 'p' denoted by
$J_p(x)$ is $J_p(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (\frac{x}{2})^{2n+p}}{n!(p+n)!}$.

17. (a) Show that the null solution of equation $x' = A(t)x$ is stable iff a positive a constant k exists ∍ $|\phi(t)| \leq k, t \geq t_0$.

    (b) The null solution of the system $x' = A(t)x$ is asymptotically stable iff $|\phi(t)| \to 0$ as $t \to \infty$.

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