STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2011 – 12)

SUBJECT CODE : 11MT/PC/MA14

M. Sc. DEGREE EXAMINATION, NOVEMBER 2011
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : CORE
PAPER : MODERN ALGEBRA
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL THE QUESTIONS: **(5 x 2 = 10)**

1. Find the number of 11-sylow subgroups in a group of order $11^2 \cdot 13^2$.
2. Prove that any field is a Euclidean domain.
3. Prove that for any prime '*p*' the polynomial $x^{p-1} -+ x^{p-2} + \cdots + x + 1$ is irreducible over the field of rational numbers.
4. Prove or disprove: Any polynomial of degree '*n*' over a skew field can have at most '*n*' roots in that skew field.
5. Define a simple extension of a field and give an example.

SECTION – B

ANSWER ANY FIVE QUESTIONS: **(5 x 6 = 30)**

6. If G is the internal direct product of $N_1, N_2, \cdots, N_n$, then prove that $N_i \cap N_j = (e)$ for $i \neq j$, and if $a \in N_i, b \in N_j$ then *ab* = *ba*.
7. Let *R* be a Euclidean ring. Then prove that any two elements *a, b* in *R* have a greatest common divisor '*d*' and it is of the form $\lambda a + \mu b$.
8. State and prove the Eisenstein criterion about irreducibility of polynomials over the field of rationals.
9. Define the splitting field of a polynomial *f(x)* over the field *F* and find the splitting field of the polynomial $x^4 + x^2 + 1$ in *Q[x]*. Find also the degree of the splitting field over *Q*.
10. If *K* is a finite extension of *F*, then prove that *G(K,F)* is a finite group and $o(G(K,F)) \leq [K:F]$.
11. Prove that the number of *p*-Sylow's subgroups of a finite group is *1+kp*.
12. If the rational number *r* is also an algebraic integer, prove that *r* must be an ordinary integer.

SECTION – C

ANSWER ANY THREE QUESTIONS : **(3 x 20 = 60)**

13. Prove that every finite abelian group is the direct product of cyclic groups.
14. (a) Define a Euclidean ring and prove that ring of Gaussian integers is a Euclidean ring.
    (b) If p is a prime number of the form $4n + 1$, then prove that $p = a^2 + b^2$, for some integers a and b.
15. Prove that the polynomial ring $F[x]$ is a Euclidean ring, proving the required results.
16. (a) Prove that an element $a \in K$ is algebraic over F if and only if $F(a)$ is a finite extension of F.
    (b) Define a simple extension. Is a simple extension a finite extension? Justify your answer.
17. State and prove the fundamental theorem of Galois Theory.