**STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086**
**(For candidates admitted from the academic year 2011-12 & thereafter)**

**SUBJECT CODE : 11MT/PE/FT24**

**M. Sc. DEGREE EXAMINATION, APRIL 2014**
**BRANCH I – MATHEMATICS**
**SECOND SEMESTER**

**COURSE : ELECTIVE**
**PAPER : FUZZY SET THEORY**
**TIME : 3 HOURS** **MAX. MARKS : 100**

## SECTION –A

**Answer all the questions:** **5×2=10**

1. Define standard operations of fuzzy union and fuzzy intersection.
2. Define height of a fuzzy relation $\mathcal{R}(X, Y)$.
3. Define Yager class of $t$-norms.
4. Define addition and multiplication of fuzzy numbers.
5. Give an application of fuzzy control.

## SECTION –B

**Answer any five questions:** **5×6=30**

6. For any $A \in \mathcal{F}(X)$, prove that $^{\alpha}A = \bigcap_{\beta<\alpha} {^{\beta}A} = \bigcap_{\beta<\alpha} {^{\beta+}A}$ .
7. Let $f: X \to Y$ be an arbitrary crisp function. Then for any $A \in \mathcal{F}(X)$, and $\alpha \in [0, 1]$, prove that
$$(i).\ {^{\alpha+}[f(A)]} = f({^{\alpha+}A})$$
$$(ii).\ {^{\alpha}[f(A)]} \supseteq f({^{\alpha}A}).$$
8. Let $(i, u, c)$ be a dual triple that satisfies the law of excluded middle and law of contradiction. Then prove that $(i, u, c)$ does not satisfy the distributive laws.
9. Find the solution of the fuzzy equation $A \cdot X = B$, where $A$ and $B$ are fuzzy numbers on $R^+$ given as follows.
$$A(x) = \begin{cases} 0 & for\ x \le 3\, and\ x > 5 \\ x-3 & for\, 3 < x \le 4 \\ 5-x & for\, 4 < x \le 5 \end{cases}$$
$$B(x) = \begin{cases} 0 & for\ x \le 12\, and\ x > 32 \\ (x-12)/8 & for\, 12 < x \le 20 \\ (32-x)/12 & for\, 20 < x \le 32 \end{cases}$$
10. Explain the methods of designing fuzzy controller.
11. Prove that the standard fuzzy intersection is the only idempotent fuzzy set intersection.
12. Express the fuzzy concept of ”Young” , “Middle aged”, “Old” persons by trapezoidal membership functions.

## SECTION –C

**Answer any three questions:** **3×20=60**

13. (a) Prove that a fuzzy set A on R is convex if and only if

$$A(\lambda x_1 + (1-\lambda)x_2) \geq \min[A(x_1), A(x_2)], \forall x_1, x_2 \in R, \lambda \in [0,1].$$

(b) Write down the features responsible for the Paradigm shift from the classical set theory. (10 + 10)

14. (a) Define (i) Fuzzy Relation (ii) Cylindrical Extension. Explain with example.

(b) Explain fuzzy binary relational equations. (10 + 10)

15. (a) Given a $t$-norm $i$ and an involutive fuzzy complement $c$, prove that the binary operation $u$ on [0, 1] defined by $u(a,b) = c(i(c(a), c(b)))$, for all $a, b \in [0,1]$, is a $t$-conorm such that $(i, u, c)$ is a dual triple.

(b) Illustrate the result given in (a) for the t-norm $i(a,b) = ab$ and the Sugeno class of fuzzy complements $c_\lambda(a) = \frac{1-a}{1+\lambda a}$ $(\lambda > -1)$. (10+10)

16. Define the basic operation of addition(+), subtraction(-), multiplication(.) and division(/) of fuzzy numbers $A$ and $B$. Further, prove that if $A$ and $B$ are continuous fuzzy numbers, prove that $A * B$ is a continuous fuzzy number, where $* \in \{+, -, ., /\}$.

17. Discuss any one application of fuzzy mathematics in industry.

⅄⅄⅄⅄⅄⅄⅄⅄⅄⅄⅄⅄