STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted from the academic year 2011-12 & thereafter)

SUBJECT CODE : 11MT/PC/TO24

M. Sc. DEGREE EXAMINATION, APRIL 2014
BRANCH I – MATHEMATICS
SECOND SEMESTER

COURSE : CORE
PAPER : TOPOLOGY
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

Answer all the questions: **5×2=10**

1. Define a Nowhere dense set.
2. Define the closure of a set A, is a topological space X.
3. Define a compact space.
4. Define a Hausdorff space.
5. Define a locally connected space.

SECTION – B

Answer any five questions: **5×6=30**

6. Show that f is continuous if and only if $f^{-1}(G)$ is open in X whenever G is open in Y where X and Y are metric spaces and $f: X \rightarrow Y$.
7. Show that the following conditions are equivalent when f is a function, $f: X \rightarrow Y$
   (i) f is continuous.
   (ii) inverse image of each basic open set is open.
   (iii) inverse image of each sub basic open set is open.
8. Show that a closed subspace of a compact space is compact.
9. Prove that a metric space is sequentially compact if and only if it has Bolzano Weierstrass property.
10. Prove that the product of any non empty class of class of Hausdorff space is Hausdorff.
11. Prove that $\bigcup_{i=1}^{\infty} A_i$ is a connected subspace of a topological space X where $\{A_i\}_{i \in I}$ a non-empty class of connected subspaces of X such that $\bigcap_{i=1}^{\infty} A_i \neq \phi$.
12. If X is a topological space the prove the following
    (i) each point in X is contained in exactly one component of X.
    (ii) each connected subspace of X is contained in a component of X.

SECTION – C

Answer any three questions: **3×20=60**

13. a) State and prove Cantors Intersection theorem.
    b) If X is a complete metric space and Y a subspace of X, then prove that Y is complete if and only if it is closed.

14. a) State and prove Lindelöf covering theorem.
    b) Show that every separable metric space is second countable.

15. State and prove Heine Borel theorem.

16. State and prove Uryshon's lemma.

17. a) Prove that the product of any non empty class of connected spaces is connected.
    b) Prove that $\mathbb{R}^n$ and $\mathbb{C}^n$ are connected.