STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2011–12 & thereafter)

SUBJECT CODE: 11MT/PC/PD44

M. Sc. DEGREE EXAMINATION, APRIL 2014
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE : CORE
PAPER : PARTIAL DIFFERENTIAL EQUATIONS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL THE QUESTIONS: (5 x 2 = 10)

1. Form the partial differential equation by eliminating arbitrary function from $z = \text{f}(\text{x} - \text{y})$.
2. When a partial differential equation $F(\, D, D'\,)Z = f(x, y)$ is said to be reducible.
3. State exterior Dirichlet problem for a circle.
4. State Maximum-Minimum principle.
5. Show that $\bar{E}$, an electric field satisfies the wave equation.

SECTION – B

ANSWER ANY FIVE QUESTIONS: (5 x 6 = 30)

6. Find the equation of the integral surface of the differential equation $2y(z-3) + \quad (2x - z)q = y(2x-3)$ which passes through the circle $z = 0, x^2 + y^2 = 2x$.
7. Show that the equations $xp - yq = x, x^2p + q = xz$ are compatible and find their solution.
8. If f and g be arbitrary functions of their respective arguments ,show that $u = f(x - vt + i\alpha y) + g(x - vt - i\alpha y)$ is a solution of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$, provided $\alpha^2 = 1 - \frac{v^2}{c^2}$.
9. Derive the Poisson's equation in the form $\nabla^2 v = -\,4\pi G\rho$
10. Find the solution of the one-dimensional diffusion equation satisfying the boundary conditions (i) T is bounded as $t \longrightarrow \infty$
    (ii) $\frac{\partial T}{\partial x}|x = 0 = \ 0$ for all t
    (iii) $\frac{\partial T}{\partial x}|x = a = \ 0$ for all t.
11. A transmission line 1000km long is initially under steady state conditions with potential 1300 volts at the sending end $(x = 0)$ and 1200 volts at the receiving end $(x = 1000)$. The terminal end of the line is suddenly grounded but the potential at source is kept at 1300 volts. Assuming the inductance and leakance to be negligible, find the potential $V(x, t)$.
12. Obtain D' Alembert solution of one-dimensional wave equation.

SECTION – C

ANSWER ANY THREE QUESTIONS: (3 x 20 = 60)

13. a) Solve the partial differential equation $z^2 = pqxy$ by charpits method.
    b) Solve $p^2x + q^2y = z$ by Jacobi's method.

14. a) Solve the equation $\frac{\partial^3 Z}{\partial x^3} - 2\frac{\partial^3 Z}{\partial x^2 \partial y} - \frac{\partial^3 Z}{\partial x \partial y^2} + 2\frac{\partial^3 Z}{\partial y^3} = e^{x+y}$
    b) Reduce the equation $(n-1)^2\frac{\partial^2 Z}{\partial x^2} - y^{2n}\frac{\partial^2 Z}{\partial y^2} = ny^{2n-1}\frac{\partial Z}{\partial y}$ to canomical form and find its general solution.

15. State and prove interior Dirichllet problem for a circle.

16. a) In the radical flow of heat through a solid sphere of radius R, the temperature T at any point distant r from the centre at any time t is given by $\frac{\partial T}{\partial t} = c^2(\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r})$ where c is a constant. Show that if $S = rT$, then S is given by $\frac{\partial S}{\partial t} = c^2\frac{\partial^2 T}{\partial r^2}$. Hence by method of separation of variables find the temperature T at any time given that (i) surface of sphere is maintained at zero temperature. (ii) $S = 0$ at the centre of sphere at all times. (iii) Initially the temperature at any point at distance r from the centre is given by $T = \frac{T_0}{r}\, sin\,(\frac{\pi s}{R})$
    b) State and prove the uniqueness theorem of heat conduction equation.

17. a) Solve the Cauchy problem, described by the in-homogeneous wave equation $\frac{\partial^2 u}{\partial t^2} - c^2\frac{\partial^2 u}{\partial x^2} = f(x,t)$ subject to initiate conditions $u(x,0) = \eta(x)$, $\frac{\partial u}{\partial t}(x,\ 0) = v(x)$, $\frac{\partial^2 u}{\partial t^2} - c^2\frac{\partial^2 u}{\partial x^2} = f(x,t)$ subject to the homogeneous initial conditions $u_2(x,0) = 0$, $\frac{\partial u_2}{\partial t}(x,0) = 0$
    b) Obtain the solution of the Radio equation $\frac{\partial^2 v}{\partial x^2} = LC\frac{\partial^2 v}{\partial t^2}$ appropriate to the case when a periodic e.m.f. V_0 cospt is applied at the end x=0 of the line.