STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2011-12 & thereafter)

SUBJECT CODE : 11MT/PC/LA24

M. Sc. DEGREE EXAMINATION, APRIL 2014
BRANCH I – MATHEMATICS
SECOND SEMESTER

COURSE : CORE
PAPER : LINEAR ALGEBRA
TIME : 3 HOURS **MAX. MARKS : 100**

Section-A
Answer ALL the questions **(5x2=10)**

1. Show that similar matrices have the same characteristic polynomial.
2. Define R-module and finitely generated R –module.
3. Prove that if $u \in V_1$ is such that $u\, T^{n_1-k} = 0$, where $0 < k \leq n_1$, then $u = u_0 T^k$ for some $u_0 \in V_1$.
4. Show that the linear operator U on V is unitary iff the adjoint $U^*$of U exists and $UU^* = U^*U = 1$.
5. Define Sesqui – linear form.

Section-B
Answer any FIVE questions **(5x6=30)**

6. Show that the characteristic and minimal polynomials for a linear operator T on n – dimensional vector space V have the same roots except for multiplicities.
7. Prove that a linear operator T is triangulable iff the minimal polynomial for T is a product of linear polynomials over F.
8. If A & B are sub modules of a module M, prove that $\frac{A+B}{B} \approx \frac{A}{A \cap B}$.
9. If $T \in A(V)$ is nilpotent of index n_1 then show that basis of V can be found such that the matrix of T in this basis has the form
$$\begin{pmatrix} M_{n_1} & 0 & \dots & 0 \\ . & M_{n_2} & \dots & 0 \\ . & \dots & \dots & \dots \\ 0 & 0 & \dots & M_{n_r} \end{pmatrix}$$
where $n_1 \geq n_2 \dots \geq n_r$ and $n_1 + n_2 \dots + n_r = dim_F V$, stating required results.

10. Prove that if $T \in A(V)$, has a minimal polynomial over F given by $p(x) = \gamma_0 + \gamma_1 x + \cdots. + \gamma_{r-1} x^{r-1} + x^r$ and further that V as a module is a cyclic module then there is a basis of V over F such that in this basis, the matrix of T is

$$\begin{pmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \ldots & \ldots & \ldots & & \\ 0 & 0 & 0 & \ldots & 1 \\ -\gamma_0 & -\gamma_1 & \ldots & & -\gamma_{r-1} \end{pmatrix}.$$

11. Prove that if T is a linear transformation from V into W, where V and W are finite dimensional inner product space with same dimension over the same fields then the following are equivalent.
    1. T preserves inner products.
    2. T is an (inner product space) isomorphism.
    3. T carries every orthonormal basis for V onto an orthonormal basis for W.
    4. T carries some orthonormal basis for V onto an orthonormal basis for W.

12. Prove that for a finite dimensional inner product space V and form f there exist a unique linear operator T on V such that $f(\alpha, \beta) = (T\alpha|\beta)$ for all α, β in V and $f \rightarrow T$ is an isomorphism.

Section-C

Answer any THREE questions (3x20=60)

13. State and prove Cayley Hamilton theorem and prove that the minimal polynomial for $T \in A(V)$ divides the characteristic polynomial for $T \in A(V)$.

14. State and prove fundamental theorem on finitely generated modules .

15. Prove that for each $i = 1,2, \ldots\ldots k, V_i \neq 0$ and $V = V_1 \oplus V_2 \oplus \ldots \oplus V_k$, the minimal polynomial of T_i is $q_i(x)^{l_i}$.

16. Prove that for every invertible complex $n \times n$ matrix B there exists a unique lower triangular matrix M with positive entries on the main diagonal such that MB is unitary.

17. Prove: There exists a subspace W of V, invariant under T, such that $V = V_1 \oplus W$.

⅄⅄⅄⅄⅄⅄⅄⅄⅄⅄⅄⅄