STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2011–12 & thereafter)

SUBJECT CODE: 11MT/PC/FA44

M. Sc. DEGREE EXAMINATION, APRIL 2014
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE	**:**	**CORE**
PAPER	**:**	**FUNCTIONAL ANALYSIS**
TIME	**:**	**3 HOURS** **MAX. MARKS : 100**

SECTION—A (5x2=10)
ANSWER ALL THE QUESTIONS

1. Define an isometric isomorphism.
2. If S_1 *and* S_2 are nonempty subsets of H such that $S_1 \subseteq S_2$ then, prove$S_1^{\perp} \supseteq S_2^{\perp}$.
3. Prove that a sequence of self-adjoint operators converges to a self-adjoint operator.
4. Does an operator T on H necessarily have an eigen value? Justify.
5. When do we say an element of a Banach algebra as topological divisor of zero?

SECTION—B (5x6=30)
ANSWER ANY FIVE QUESTIONS

6. State and prove Minkowski's inequality.
7. State and prove Banach-Steinhaus theorem.
8. Prove that a closed convex subset C of a Hilbert space H contains unique vector of smallest norm.
9. Prove that every non-zero Hilbert space contains a complete orthonormal set.
10. Prove that an operator T on H is unitary if and only if it is an isometric isomorphism of H onto itself.
11. If T is an normal operator on H, then the eigen spaces M_i are pairwise orthogonal.
12. Define spectrum of x and show that it is non-empty.

SECTION—C (3x20=60)
ANSWER ANY THREE QUESTIONS

13. State and Prove the Open mapping theorem.
14. State and Prove Bessel's inequality.
15. Let H be a Hilbert space, and let f be an arbitrary functional in the conjugate space H^*. Then prove that there exists a unique vector y in H such that $f(x) = (x, y)$ for every x in H. Also prove that the mapping is an isometry.
16. Two matrices in A_n are similar if and only if they are the matrices of a single operator on H relative to different bases.
17. a) Prove that the set of all invertible elements G of a Banach algebra A is an open set and also prove that the set of all singular elements S of A is a closed set.

    b) State and prove the properties of topological divisors of zero.