STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2011-12 & thereafter)

SUBJECT CODE : 11MT/PC/DG44

M. Sc. DEGREE EXAMINATION, APRIL 2014
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE : CORE
PAPER : DIFFERENTIAL GEOMETRY
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL QUESTIONS : **(5 X 2 = 10)**

1. Define centre of curvature.
2. Define orientable surface.
3. Define an isometry.
4. Find the second fundamental form of the plane $\sigma(u, v) = a + up + vq$.
5. Define Gaussian and Mean curvature.

SECTION – B

ANSWER ANY FIVE QUESTIONS : **(5 X 6 = 30)**

6. Define regular point and prove that any reparametrisation of a regular curve is regular.
7. Show that a parametrised curve has a unit-speed reparametrisation if and only if it is regular.
8. Show that the level surface $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, where a, b and c are non-zero constants, is a smooth surface.
9. Find the first fundamental form for $\sigma(u, v) = (sinhu\ sinhv, sinhu\ coshv, sinhu)$.
10. Prove that a diffeomorphism $f: S_1 \to S_2$ is an isometry if and only if, for any surface patch σ_1 of S_1, the patches σ_1 and $f \circ \sigma_1$ of S_1 and S_2, respectively, have the same first fundamental form.
11. State and prove Euler's theorem.
12. Find the Gaussian and mean curvature of the surface $\sigma (u, v) = (u + v, u - v, uv)$ at the point (2 , 0, 1).

SECTION – C

ANSWER ANY THREE QUESTIONS : (3 X 20 = 60)

13. (a) Let $\gamma(t)$ be a regular curve in $\mathbb{R}^3$. Prove that its curvature is

$$k = \frac{\|\ddot{r} \times \dot{r}\|}{\|\dot{r}\|^3}$$

where the $\times$ indicates the vector (or) cross product and the dot denotes $\frac{d}{dt}$.

(b) Obtain the Frenet- serret equations. (10+10)

14. Prove that the unit sphere and the double cone are surfaces.

15. Prove that a diffeomorphism$f: S_1 \to S_2$ is conformal if and only if, for any surface patch $\sigma_1$on S_1, the first fundamental forms of σ_1 and $f \circ \sigma_1$proportional.

16. Let k_1 and k_2 be the principal curvatures at a point P of a surface patch σ.Then show that

(i) k_1 and k_2 are real numbers;

(ii) if $k_1 = k_2 = k$, say, then $\mathcal{F}_{II} = k\mathcal{F}_1$ and (hence) every tangent vector to σ at P is a principal vector;

(iii) if $k_1 \neq k_2$, then any two (non-zero) principal vectors t_1 and t_2 corresponding to k_1 and k_2, respectively, are perpendicular.

17. State and prove Gauss's remarkable theorem.