STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2008–09 & thereafter)

SUBJECT CODE : MT/ME/FM54

B. Sc. DEGREE EXAMINATION, NOVEMBER 2011
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE	**:**	**MAJOR – ELECTIVE**	
PAPER	**:**	**FINANCIAL MATHEMATICS**	
TIME	**:**	**3 HOURS**	**MAX. MARKS : 100**

Answer Any Six Questions (each carrying 17marks)

1. (a) Prove that the Geometric Brownian motion is a limit of simpler models. **(14)**
   (b) Prove that $Cov(X_1 + X_2, Y) = Cov(X_1, Y) + Cov(X_2, Y)$. **(3)**

2. (a) An individual who plans to retire in 20 years has decided to put an amount *A* in the bank at the beginning of each of the next 240 months, after which she will withdraw $1000 at the beginning of each of the following 360 months. Assuming a nominal yearly interest rate of 6% compounded monthly, how large does *A* need to be? **(5)**
   (b) Define the effective interest rate and explain the Doubling Rule method. **(5)**
   (c) Suppose that you are to receive payments (in thousands of dollars) at the end of each of the next five years. Which of the following three payment sequences is preferable if the interest rate compounded annually at 10%?
   A : 12, 14, 16, 18, 20 **B** : 16, 16, 15, 15, 15 **C** : 20, 16, 14, 12, 10. **(7)**

3. (a) State and prove the Law of One Price and illustrate the same by an example. **(12)**
   (b) Let $C(K, t)$ be the cost of a call option on a specified security that has strike price *K* and expiration time *t*. Prove that for fixed expiration time *t*, $C(K, t)$ is a convex in *K* and non-increasing function of *K* **(5)**

4. (a) State and prove Arbitrage Theorem. **(10)**
   (b) Let the initial price of a stock be $100 and the price after one period is assumed to be either $200 or $50. At a cost of C per share, you can purchase at time 0 the option to buy the stock at time 1 for the price of $150. For what value of C is no sure win possible? **(7)**

5. State and prove properties of the Black-Scholes option cost. **(17)**

6. (a) Prove that the amount of money needed at time 0 is equal to the expected present value, under the risk-neutral probabilities of the payoff at time 1, by delta hedging arbitrage strategy. **(10)**

   (b) With usual notation, prove that $e^{-rt}E[IS(t)] = s\phi(\omega)$. **(7)**

7. (a) Describe the Portfolio Selection Problem and solve it. **(10)**

   (b) An investor with capital *x* can invest any amount between 0 and *x*; if *y* is invested then *y* is either won or lost, with respective probabilities *p* and 1 - *p*. If $p > 1/2$, how much should be invested by an investor having a log utility function? **(7)**

8. (a) How do you value investments by expected utility? **(10)**

   (b) Explain Barrier, Asian and Look back Options. **(7)**